

Gravitation and Satellites Assignment 1 SOLUTIONS

1. a) $m_1 = 3.5 \times 10^5 \text{ kg}$ $m_2 = 8.9 \times 10^{23} \text{ kg}$ $r = 4.1 \times 10^5 \text{ km} = 4.1 \times 10^8 \text{ m}$

$$F = G \frac{m_1 m_2}{r^2}$$
$$= 6.67 \times 10^{-11} \frac{3.5 \times 10^5 \times 8.9 \times 10^{23}}{(4.1 \times 10^8)^2}$$
$$= 1.2 \times 10^2 \text{ N}$$

Serenity experiences $1.2 \times 10^2 \text{ N}$ (2 s.f.) towards Rigel II.

- b) $1.2 \times 10^2 \text{ N}$ (2 s.f.) towards Serenity.

- c) $m_1 = 3.5 \times 10^5 \text{ kg}$ $F = 1.2 \times 10^2 \text{ N}$

$$F = m_1 a$$

$$\therefore a = \frac{F}{m_1}$$

$$= \frac{1.2 \times 10^2 \text{ N}}{3.5 \times 10^5 \text{ kg}}$$

$$= 3.5 \times 10^{-4} \text{ ms}^{-2}$$

Serenity accelerates at $3.5 \times 10^{-4} \text{ ms}^{-2}$ (2 s.f.) towards Rigel II.

- d) There will be no effect. Gravitational acceleration only depends on the *other* body's mass, according to $g = G \frac{M}{r^2}$

2. Some possible options:

Geostationary	Polar
High orbit	Low orbit
Long period	Short period
East-west or west-east direction	Close to north-south or south-north direction
Slower speed	Higher speed

3.

a) Equating $v = \sqrt{\frac{GM}{r}}$ and $v = \frac{2\pi r}{T}$ gives:

$$\frac{2\pi r}{T} = \sqrt{\frac{GM}{r}}$$

$$\therefore \left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r}$$

$$\therefore \frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

$$\therefore 4\pi^2 r^3 = T^2 GM \quad \{\text{multiplying both sides by } T^2 r\}$$

$$\therefore r^3 = \frac{T^2 GM}{4\pi^2}$$

$$\therefore r = \sqrt[3]{\frac{T^2 GM}{4\pi^2}}$$

b) $M = 5.97 \times 10^{24} \text{ kg}$ $R = 6.4 \times 10^6 \text{ m}$ $T = 24 \text{ hours} = 8.64 \times 10^4 \text{ s}$ $r = ?$

$$r = \sqrt[3]{\frac{T^2 GM}{4\pi^2}}$$

$$= \sqrt[3]{\frac{(8.64 \times 10^4)^2 \times 6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{4\pi^2}}$$

$$= 4.22 \times 10^7 \text{ m (3 s.f.)}$$

So a geostationary satellite is at $4.22 \times 10^7 \text{ m}$ from the centre of the Earth.

The altitude is $4.22 \times 10^7 - 6.4 \times 10^6 = 3.6 \times 10^7 \text{ m}$ (2 s.f.) from the Earth's surface.

4.

$$F = G \frac{m_1 m_2}{r^2}$$

G, m_1 and m_2 are constant.

$$\therefore F \propto \frac{1}{r^2}$$

$\therefore F$ and $\frac{1}{r^2}$ are in constant ratio

$$r_A = r, \quad r_B = 3r$$

$$\therefore \frac{F_A}{F_B} = \frac{\frac{1}{r_A^2}}{\frac{1}{r_B^2}} = \frac{1}{r_A^2} \times \frac{r_B^2}{1} = \frac{r_B^2}{r_A^2}$$

$$\therefore \frac{F_A}{F_B} = \frac{(3r)^2}{r^2} = \frac{9r^2}{r^2} = \frac{9}{1}$$

$$\therefore F_B = \frac{F_A}{9} = \frac{195}{9} = 21.7 \text{ N}$$

Optional revision solutions

1.

a) (one of various possible methods)

$$t = 18.7\text{s} \quad s_H = 1.98 \times 10^3\text{m} \quad v_0 = 140\text{ms}^{-1} \quad \theta = ?$$

Follow the usual process for finding the range from time of flight, but use the pronumeral θ for the angle. Then rearrange to find θ .

$$v_H = v_{0_H} = 140 \cos \theta$$

$$s_H = v_H t$$

$$\therefore 1.98 \times 10^3 = 140 \cos \theta \times 18.7$$

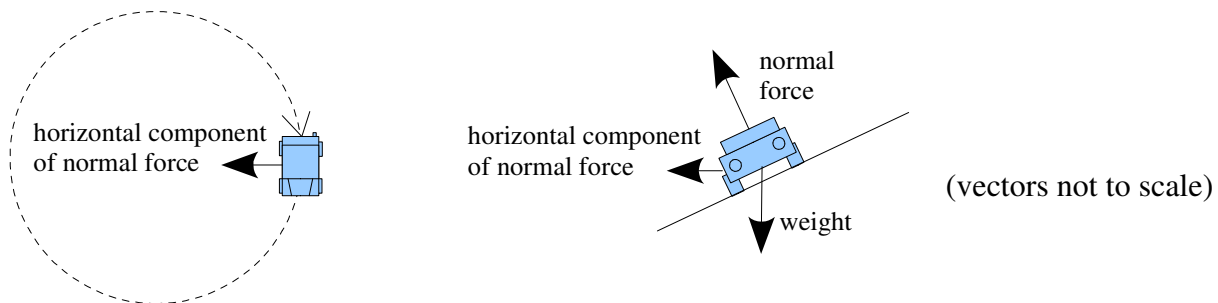
$$\therefore \cos \theta = \frac{1.98 \times 10^3}{140 \times 18.7}$$

$$\therefore \theta = \cos^{-1} \left(\frac{1.98 \times 10^3}{140 \times 18.7} \right)$$
$$= 40.8^\circ \text{ (3 s.f.)}$$

- b) 49° (assuming the projectile is fired on level ground) – because the max range will be at 45° so 4° either side of this will be the same range (some range less than max).
- c) The time of flight would be longer. The angle being higher means the vertical component of initial velocity would be larger, which increases time of flight.

2.

a)



The car is relying only on the normal force to accelerate, and the normal force is unchanged by the icy surface of the road (this only affects friction). Therefore, the car will continue to turn the corner unaffected.

- b) (i) If the road was banked less steeply, the car would be relying on friction to provide some of the centripetal force. If the road became icy, the force of friction would be less, so the force necessary to accelerate into the corner would not be provided; the radius of the car's turn would increase (the car would go off the outside of the corner).
- (ii) If the road was banked more steeply, the car would be relying on friction to stop it sliding down the banking. If the road became icy, this friction would no longer be provided, and the car would slide towards the inside of the corner.