Momentum in Two Dimensions Assignment 1 ***SOLUTIONS***

1. a) $p = mv$

$\therefore p = 0.2 \times 12$

$= 2.4\ \text{kgms}^{-1}$

b) $F = \frac{\Delta p}{\Delta t} = \frac{2.4}{6 \times 10^{-3}} = 400\text{N}$ (1 s.f.) to the right

c) 400N (1 s.f.) to the left

$\vec{F}_1 = -\vec{F}_2$

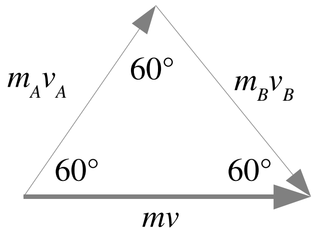
2. a) The total momentum of any number of objects remains unchanged in the absence of external forces.

b) Each expelled exhaust letter is given momentum – but the total rocket-letters system cannot have changed its momentum, so the ship is given momentum in the opposite direction to cancel it out. Since this is happening at a constant rate, the ship has constant acceleration.

c) Final momentum must add vectorially to give initial momentum, so we can make a vector diagram as to the right.

$\vec{p}_{rocket} = \vec{p}_A + \vec{p}_B$

$\therefore m\vec{v} = m_A\vec{v}_A + m_B\vec{v}_B$



NOTE: This is definitely not the only way to solve this problem. Other ways include dividing the triangle into two right angled trangles or using the cosine rule.

All angles of the triangle are 60° (since we know the first two angles and there is a total of 180° in a triangle) which means it is equilateral. So the lengths of the sides are the same, so we have $m_A v_A = mv$

$\therefore \frac{m}{2} v_A = mv$

$\therefore v_A = 2v \quad \{\text{multiplying both sides by } \frac{2}{m}\}$

3. $\vec{F} = m\vec{a}$

$\therefore \vec{F} = m\frac{\vec{v}_f - \vec{v}_i}{\Delta t}$

$\therefore \vec{F} = \frac{m\left(\vec{v}_f - \vec{v}_i\right)}{\Delta t}$

$\therefore \vec{F} = \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t}$

$\therefore \vec{F} = \frac{\Delta m\vec{v}}{\Delta t}$

$\therefore \vec{F} = \frac{\Delta\vec{p}}{\Delta t}$

Momentum in Two Dimensions Assignment 2 ***SOLUTIONS***

1. a) $\vec{F}_1 = -\vec{F}_2 \quad \{\text{Newton's third law}\}$

$$\therefore \frac{\Delta\vec{p}_1}{\Delta t} = -\frac{\Delta\vec{p}_2}{\Delta t} \quad \left\{\vec{F} = \frac{\Delta\vec{p}}{\Delta t}\left(\text{Newton's second law in terms of momentum}\right)\right\}$$

$$\therefore \Delta\vec{p}_1 = -\Delta\vec{p}_2$$

$$\therefore \Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$$

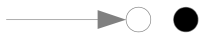
b) $\Delta\vec{p}_1 + \Delta\vec{p}_2 + \Delta\vec{p}_3 = 0$

2. 100gs^{-1} (100g per second) is 0.100kg every 1.00 seconds.

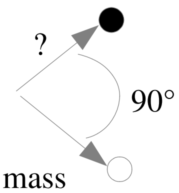
So $m = 0.100\text{kg}$ and $\Delta t = 1.00\text{s}$

$$F = \frac{\Delta p}{\Delta t} = \frac{mv}{\Delta t} = \frac{0.100 \times 10}{1.00} = 1.0\text{N}$$

3. Before



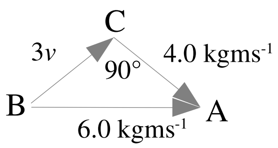
After



Total momentum P = mv = $2.0 \times 3.0 = 6.0\text{kgms}^{-1}$ to the right

Momentum of 2kg mass
P = mv = $2.0 \times 2.0 = 4.0\text{kgms}^{-1}$

Total momentum before = total after so can make a vector diagram:



$$6^2 = 4^2 + (3v)^2 \quad \{\text{pythagoras}\}$$

$$\therefore 9v^2 = 6^2 - 4^2$$

$$\therefore v = \sqrt{\frac{6^2 - 4^2}{9}} = 1.5$$

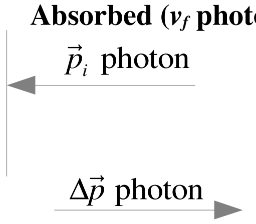
The speed of the 3.0 kg mass after the collision is $1.5\ \text{ms}^{-1}$

4.

a) Light has momentum. If light hits the sail and is absorbed or reflected then it experiences a change in momentum. Since momentum must be conserved, the ship will change momentum to make up the difference. This leads to acceleration since the mass of the sail is constant so momentum is changed by change in velocity.

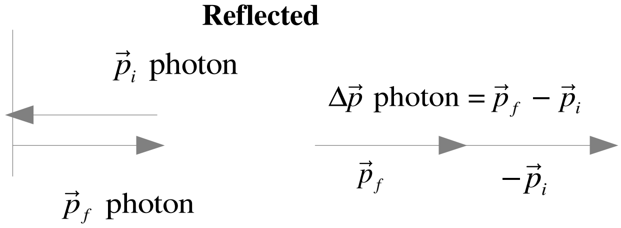
b)

Absorbed (v_f photon = 0)



$$\vec{F} \text{ on craft} = -\frac{\Delta\vec{p}\text{ photon}}{\Delta t}$$

Reflected



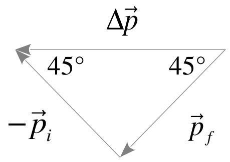
$$\vec{F} \text{ on craft} = -\frac{\Delta\vec{p}\text{ photon}}{\Delta t}$$

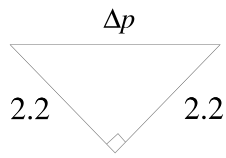
A reflected photon's velocity changes more than an absorbed photon, so more force is imparted to conserve momentum. Reflection will impart more force, and since F = ma it will therefore impart more acceleration.

Momentum in Two Dimensions Assignment 3 ***SOLUTIONS***

1.

$\Delta\vec{p} = \vec{p}_f - \vec{p}_i$ $\qquad$ $p = mv = 0.53\times4.1 = 2.2\ \text{kgms}^{-1}$





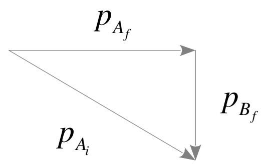
$$\Delta p = \sqrt{2.2^2 + 2.2^2} = 3.1\ \text{kgms}^{-1}$$

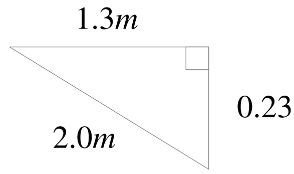
The change in momentum is 3.1 kgms^{-1} to the left (away from the wall).

2. According to the law of conservation of momentum, total final momentum will be equal to total initial momentum (change in total momentum is zero).

$$\vec{p}_{T_f} = \vec{p}_{T_i}$$
$$\therefore \vec{p}_{A_f} + \vec{p}_{B_f} = \vec{p}_{A_i} + \vec{p}_{B_i}$$
$$\therefore \vec{p}_{A_f} + \vec{p}_{B_f} = \vec{p}_{A_i} \quad \{\text{since ball B is stationary initially}\}$$

$$p_{A_i} = m_A v_{A_i} = m\times2.0\ \text{kgms}^{-1}$$
$$p_{A_f} = m_A v_{A_f} = m\times1.3\ \text{kgms}^{-1}$$
$$p_{B_f} = m_B v_{B_f} = 0.25\times0.9 = 0.23\ \text{kgms}^{-1}$$



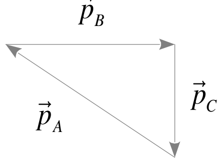


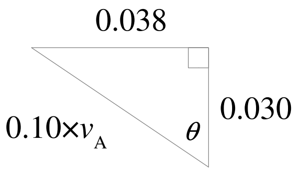
$$(2.0m)^2 = (1.3m)^2 + (0.23)^2$$
$$\therefore 4.0m^2 = 1.69m^2 + 0.053$$
$$\therefore 2.31m^2 = 0.053$$
$$\therefore m^2 = 0.023$$
$$\therefore m = 0.15\ \text{kg}$$

3. According to the law of conservation of momentum, total final momentum will be equal to total initial momentum (in this case, zero).

$$\vec{p}_{T_f} = \vec{p}_{T_i}$$
$$\therefore \vec{p}_A + \vec{p}_B + \vec{p}_C = 0$$

$$p_A = m_A v_A = 0.10\times v_A\ \text{kgms}^{-1}$$
$$p_B = m_B v_B = 0.13\times0.29 = 0.038\ \text{kgms}^{-1}$$
$$p_C = m_C v_C = 0.095\times0.32 = 0.030\ \text{kgms}^{-1}$$





$$0.10\times v_A = \sqrt{0.038^2 + 0.030^2} \qquad \theta = \tan^{-1}\left(\frac{0.038}{0.030}\right) = 51°$$
$$\therefore v_A = \frac{\sqrt{0.038^2 + 0.030^2}}{0.10}$$
$$\therefore v_A = 0.48\ \text{ms}^{-1}$$

The final speed and direction of fragment A is 0.48 ms^{-1} at 129° clockwise of the direction fragment C travels.