Projectile Motion

An object under projectile motion experiences constant acceleration downwards due to gravity and follows a parabola.

A projectile follows these vector equations:

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$\vec{s} = \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2$$

To perform calculations, one dimensional equations for each component are used:

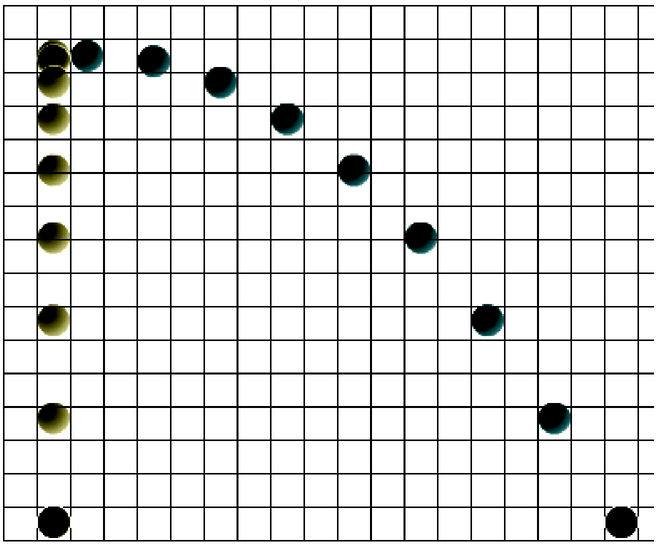
Displacement equations	$s_H = v_{0_H} t + \frac{1}{2} a_H t^2$ (this simplifies to $s_H = v_{0_H} t$ or $s_H = v_H t$) $s_V = v_{0_V} t + \frac{1}{2} a_V t^2$
Velocity equations	$v_H = v_{0_H} + a_H t$ (this simplifies to $v_H = v_{0_H}$ since $a_H = 0$) $v_V = v_{0_V} + a_V t$

Don't forget the sign where necessary.

Vertical and Horizontal Components of Velocity

A projectile's velocity is a vector and can be resolved into two components (horizontal $\vec{v}_H$ and vertical $\vec{v}_V$). These components are vectors at right angles to each other.

A multi-image photograph can be used to demonstrate the behaviour of the components.

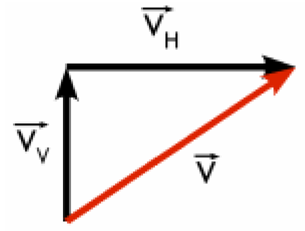


In a multi-image photograph, the time difference between images is constant.
Since the projectile moves the same horizontal distance every time, its horizontal component of velocity must be constant ($s_H = v_H t$).

Both objects have zero initial vertical component of velocity to begin with. They fall to the ground in the same amount of time, which shows:

- horizontal motion and vertical motion are independent of each other
- projectiles have the same acceleration as a vertically free-falling object

The velocity vector at any instant can be found by adding the components vectorially:



In practice, this can be done using calculations or a scale diagram.

Since the components are at right angles, they act independently.
In the absence of air resistance:

- vertical component has constant acceleration (in direction of gravitational force)
- horizontal component has constant horizontal speed

The component properties can be shown by drawing vectors on the projectile at any point of its motion.

Determination of the Vertical Component of Velocity

Given the initial velocity $\vec{v}_0$ of a projectile as:

- initial speed v_0
- and direction θ to the horizontal

Calculate the magnitude of the vertical component of velocity using $v_v = v\sin\theta$ (insert v_0 for v).

Don't forget: Components are vectors.

The **vertical** component of velocity acts according to these one dimensional motion equations:

$$v_V = v_{0_V} + a_V t$$
$$v_V{}^2 = v_{0_V}{}^2 + 2a_V s_V$$

To calculate the vertical component of velocity at any instant *t*, insert the necessary values into
$v_V = v_{0_V} + a_V t$
Remember: these are one dimensional equations.

Resolution of Velocity into Components

Given the velocity vector $\vec{v}$ of a projectile at any instant as:

- speed *v*
- and direction θ to the horizontal

The velocity vector can be resolved into its horizontal and vertical components:

- The magnitude of the horizontal component of velocity $v_H = v\cos\theta$
- The magnitude of the vertical component of velocity $v_v = v\sin\theta$

Don't forget: Components are vectors.

Time of Flight

At the start of projectile motion ($t = 0$), the object is at its initial height ($s_V = 0$).
The time of flight is the value of *t* when the object reaches its final height.

The time of flight depends only on the vertical component, and:

- if s_V is known, calculate by rearranging $s_V = v_{0_V} t + \frac{1}{2} a_V t^2$
- if v_V and v_{0_V} are known, calculate by rearranging $v_V = v_{0_V} + a_V t$

Discard zero or negative values of *t*.

Range

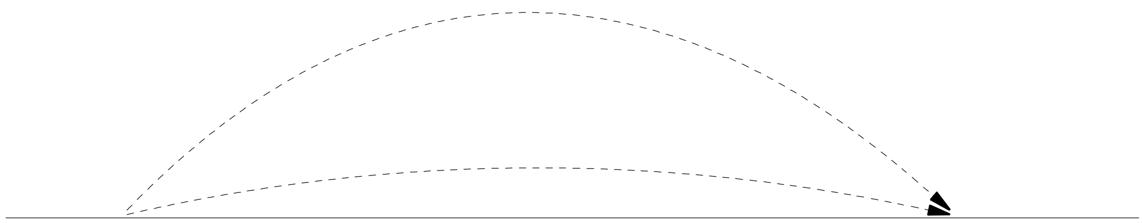
The range of a projectile is the horizontal displacement at the time of flight.
The range can be calculated using $s_H = v_{0_H} t + \frac{1}{2} a_H t^2$.
There is no horizontal acceleration ($a_H = 0$), so v_H is constant ($v_H = v_{0_H}$).

This means the formula simplifies to $s_H = v_H t$

As the simplified formula shows, the range depends on the time of flight and the horizontal component of velocity.
Launch angle and range are not proportional. Starting at θ= 0 and increasing the angle will increase the range, until the angle for maximum range is reached. Increasing the angle further will decrease the range.

- The launch angle that results in maximum range is different for every launch height
- For each launch height, there are two launch angles that result in the same range

High launch angle has longer time of flight but slower horizontal velocity



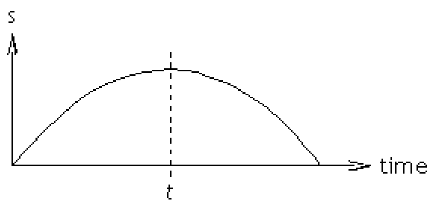
Low launch angle has shorter time of flight but faster horizontal velocity

Maximum Height

The maximum height is the launch height plus the maximum vertical displacement, and depends only on the vertical component.

At maximum height, the vertical component of velocity v_V is zero. It has finished moving upwards, and it's about to move downwards.

If the projectile's final height is the same as the initial height, the maximum height occurs when *t* is half of the time of flight (as the path of motion is symmetrical).



Effect of Air Resistance

Air resistance is a force acting in the opposite direction to the velocity of a projectile.
With air resistance, the projectile:

- will not travel as high because the vertical component of velocity is slowed down
- will have a shorter time of flight because it does not go as high
- will have a shorter range since it has a shorter time of flight **and** its horizontal component of velocity is slowed down ($s_H = v_H t$)
- will no longer follow a parabolic path, since the object is going slower at the end than the start

The force of air resistance can be explained by Newton's third law. As the object moves it pushes air particles out of the way, and they push back.
The force of air resistance is greater:

- for shapes that have a greater projected area
- for larger objects
- for faster moving objects
- for objects with rougher surface texture
- in more dense air

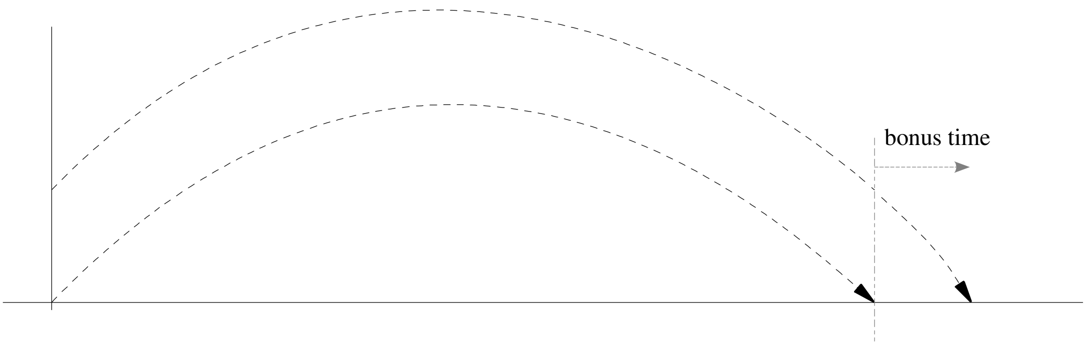
If the object pushes more air out of the way, more air is pushing back. If it pushes through the air faster, each particle of air is pushing back harder. In each case, the force on the object is greater.

Changing the mass of the object does not change the force of air resistance, since changing the mass doesn't change the amount of air pushed, so it doesn't change the force of the air pushing back.

Changing the mass *does* change the amount of acceleration, since $\vec{a} = \frac{\vec{F}}{m}$

Application: Projectiles in Sport

For a number of sports, the aim is to to obtain the maximum range.
For these sports, it is important to know which launch angle will give the maximum range for the launch height.



As the launch height increases, the maximum range increases. This is because the time of flight is increased and $s_H = v_H t$.

The angle needed for maximum range is smaller when the initial height is higher. An explanation:

- As the launch height increases, the time of flight increases.
- Increasing the horizontal component of velocity (lowering the angle) means this extra time will provide more horizontal movement.

Too much horizontal and not enough vertical will still reduce range, since the time of flight will be shortened.