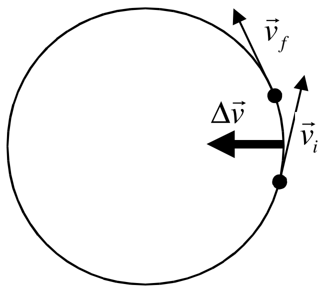
Uniform Circular Motion

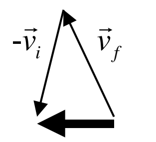
Centripetal Acceleration

Velocity in two dimensions is a vector, so it changes if either its direction or magnitude change.
An object moving in uniform circular motion (constant speed and radius) is experiencing acceleration since its direction is changing, even though its speed is not.

At any instant in time, the acceleration during uniform circular motion is perpendicular to the velocity, since the speed does not change. We can check this by using vector subtraction over a small amount of time Δt and thinking about what it would look like if it was an instant.

Take an object revolving in uniform circular motion. At one point it has velocity $\vec{v}_i$, Δt later it has velocity $\vec{v}_f$. Change in velocity is $\Delta\vec{v}$.





$\Delta\vec{v} = \vec{v}_f - \vec{v}_i$

The average acceleration $\left(\vec{a}_{\text{ave}} = \frac{\Delta\vec{v}}{\Delta t}\right)$ is in the same direction as the change in velocity, perpendicular to the velocity halfway between $\vec{v}_i$ and $\vec{v}_f$.
If Δt was infinitely small, the two velocities would occur at almost the same position. There would effectively be one velocity with acceleration exactly perpendicular to it; towards the centre.

This is true all the way around the circle of motion. The acceleration at any instant is perpendicular to the velocity, so it is directly towards the centre of the circle.

The magnitude of the acceleration is constant around the circle of motion, and given by $a = \frac{v^2}{r}$.

Period of Motion

The period T is the time the object takes to travel the circumference once.

speed = $\frac{\text{distance}}{\text{time}}$

$\therefore v = \frac{\text{circumference}}{T}$

circumference $= 2\pi r$

$\therefore v = \frac{2\pi r}{T}$

Force Causing the Centripetal Acceleration

According to Newton's 2nd Law of Motion, $F = ma$. Inserting the equation for the magnitude of centripetal acceleration gives the equation for the magnitude of the centripetal force: $F = \frac{mv^2}{r}$

The direction of the force will be the same as the direction of the acceleration.

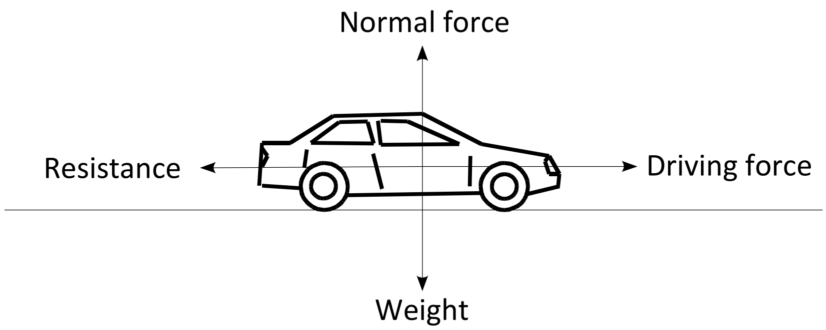
Examples of Force Causing the Centripetal Acceleration

Tension force – *Mass on a string*
A mass being spun on the end of a length of string experiences uniform circular motion. The string can only stretch so far, and then it provides a force pulling the mass in towards the centre. If the string is unable to provide this force (i.e. it breaks) then the mass will launch off tangentially, no longer in uniform circular motion.

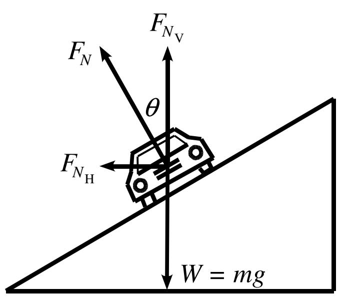
Frictional force – *Car going around a bend*
In order to navigate on a road, a car turns its tyres. Doing so angles them to the direction of motion, and they experience friction as they push against the road. If the tyres grip well enough (i.e. there is enough friction) then the force returned by the road (following Newton's 3rd law) will push the car towards the centre of the curve – it experiences circular motion. If the tyres are unable to provide the necessary friction, the car slides off the road tangentially.

Gravitational force – *Satellites*
Planets and satellites are approximate examples of uniform circular motion. Their speed is constant as there is no friction in space, and they are kept in uniform circular motion by the gravitational force towards the centre of the body they are orbiting.

Normal force - *Banked curve*
On a flat road at a **constant velocity** a car experiences a number of forces but all are being cancelled out:



On a flat road the normal force exactly cancels the weight of the car. If the road is banked, the normal force is no longer straight up. This situation is shown below.

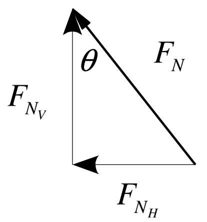


If the car were to turn a curve, on a flat road the friction of the tyres pushing the road would be needed to obtain the centripetal force, however now the horizontal component of the normal force provides some or all of the centripetal acceleration.

The vertical component must still be sufficient to keep the car from sinking into the road, so $F_{N_V} = mg$.

For the horizontal component to provide exactly all the centripetal acceleration (friction of the tyres not needed) $F_{N_H} = F_c = ma_c = m\frac{v^2}{r}$

The total normal force is the vector sum of its components, so:



$$\tan\theta = \frac{F_{N_H}}{F_{N_V}}$$

$$\therefore \tan\theta = \frac{m\frac{v^2}{r}}{mg}$$

$$\therefore \tan\theta = \frac{v^2}{rg}$$