Electric Fields

Coulomb's Law

The force two charged bodies exert on each other:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

F is the magnitude of the electric forces (measured in newtons, N)

q_1 and q_2 are the charges (measured in coulombs, C)

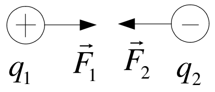
r is the distance between their centres (measured in metres, m)

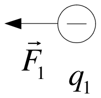
and $\frac{1}{4\pi\varepsilon_0}$ is the proportionality constant for a vacuum, with a value of 9.00×10^9

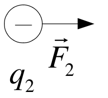
The force will be repulsion if the bodies have a like charge, and attraction if unlike.

Electric forces are consistent with Newton's third law

Each body pulls (or pushes) the other with the same magnitude of force in opposite directions, meaning that the forces are equal and opposite.







Proportionality of Coulomb's Law:

$F \propto q_1$ and $F \propto q_2$

F is said to be directly proportional to each of the charges. For example if you double one of the charges, the force between the charges will double. Halving a charge will halve the force.

$$F \propto \frac{1}{r^2}$$

F is said to be inversely proportional to the radius squared. For example if you double the distance between the charges, the force between them will be a quarter of the original force.

Principle of Superposition

When more than two point charges are present, the force on any one of them is equal to the vector sum of the forces due to each of the other point charges.

So to find the force on one charge near two others:

1) Calculate the force due to the first 'other' charge
2) Calculate the force due to the second 'other'
3) Add the two forces vectorially.

Electric Fields

The concept of an electric field replaces the concept of "action at a distance" used in Coulomb's law with the "localised action" at some point in a field.

That is, rather than thinking of the charges 'communicating' with each other in order to attract or repel, think of it as one charge setting up a field and the other experiencing the effects of being in that field.

Electric charges establish an electric field *E* in the surrounding space. The electric field at any point produces a force on an electric charge placed at that point.

Using Coulomb's Law and the definition of the electric field above we can predict the magnitude of the electric field strength some distance r from a charge q. To do this we will use a test charge Q.

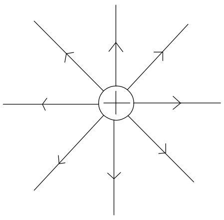
$$E = \frac{F}{Q} \quad \{\text{consider only magnitudes, figure out direction from a diagram if necessary}\}$$

$$\therefore E = \frac{\frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}}{Q} \quad \left\{\text{inserting Coulomb's Law } F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\right\}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad \{\text{the magnitude of the test charge is irrelevant as it cancels out}\}$$

Pictorial Representation of Electric Fields

An electric field can be represented by field lines such that the direction of the field is at a tangent to each line, and the magnitude of the field at any point is represented by how close the lines are together at any point. For example:



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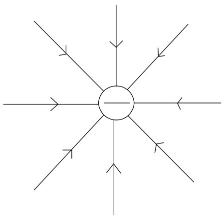
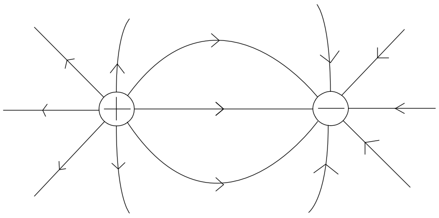


Superposition of Electric Fields

The principle of superposition applied to forces, so since electric field is force per unit charge, the principle applies to electric fields as well.

That is, the field strength vectors at any point will add vectorially, so for example if at some point there would be two field lines meeting going opposite directions, instead they would cancel out and no field line would be shown there. For example:



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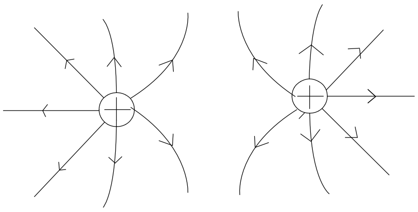
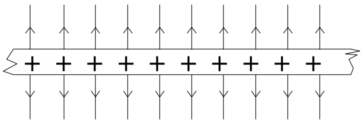
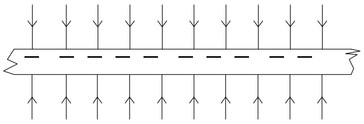


Electric Field Due to One or Two Charged Plates

The electric field due to one infinitely long charged conducting plate is uniform (that is, the field lines are equally spaced and parallel at all points). When the plate is finite, however, the field is *not* uniform on the ends (the ends act in a way similar to point charges).



Electric field due to an infinitely long positively charged conducting plate

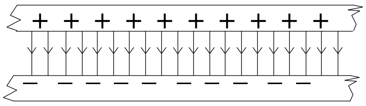


Electric field due to an infinitely long negatively charged conducting plate

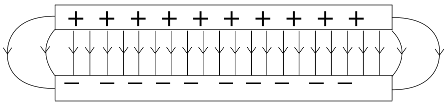
If we put one plate above the other, the electric field lines:

- between the plates are going in the same direction,
- below each plate are going opposite directions,
- above each plate are going in opposite directions

According to the principle of superposition, this means that the field strength magnitudes in the centre will add and the field strength magnitudes on the outside will subtract (the distances between the two bottoms/tops of plates is irrelevant as the fields are uniform). If the plates have equal magnitude charges, we will have twice the field between the plates and zero field on the outside.



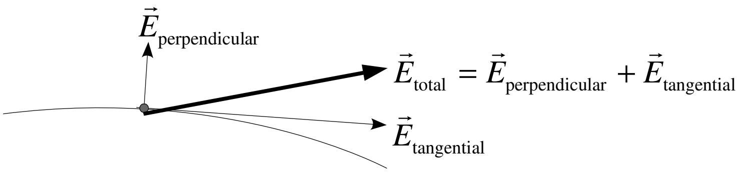
Note that *if the plates are not of infinite length*, the electric field near and beyond the edges of two finite plates is non-uniform:



Electric Fields and Conductors

Electric fields always meet the conducting surface at right angles.

Charges on a conductor are always in electrostatic equilibrium (that is they have spread out so that they are as far away from each other as possible, and hence do not move around) which means they should not move around on the surface of the conductor.



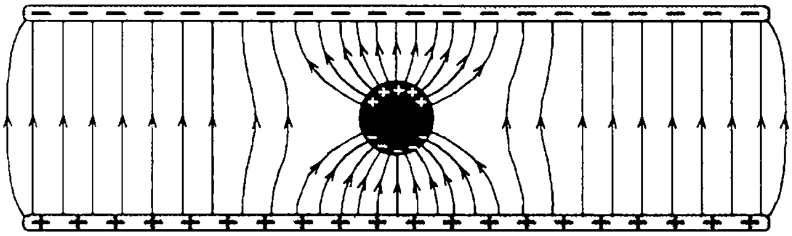
The electric field represents the force a charge feels – so if there were a component tangential to the surface then the charge will accelerate along the surface. This should not happen due to electrostatic equilibrium, so $\vec{E}_{\text{tangential}}$ must be zero. Therefore all the electric field is perpendicular.

There is no electric field inside conducting material

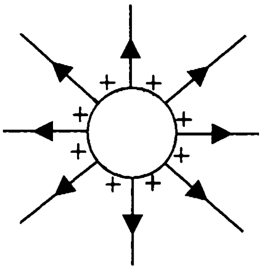
Whether the conductor is solid or not any excess charges will repel each other and move to the outside of the object. The inside of conductor reaches electrostatic equilibrium, so the charges inside should not be moving around. The only way they can be not moving around is if the field inside is zero, so this must be the case.

An uncharged conductor in an external electric field will experience charge polarisation.

That is, the conductor's delocalised (free) electrons will be forced in the opposite direction to the field lines, so the conductor gains a net positive charge on one side and a net negative charge on the other.

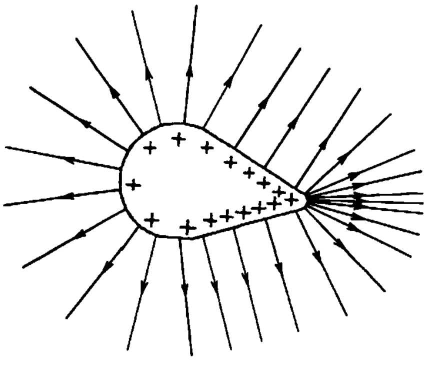


There is no electric field inside a hollow conductor of any shape, whether or not the conductor is charged, provided that there is no charge in the cavity.



Electric fields are strongest near sharp points on conductors.

This is because around tighter curves not as much of the force on any one charge is directed along the surface, so it is able to be closer to the other charges (since the only force which moves the charge is that along the surface). Also contributing to this effect (on the pointed conductor shown below) is the fact that there are charges on the left of the conductor which push the charges still further down onto the point.



Corona Discharge

A sharp point on a conductor can cause such high electric field strength that air molecules polarise and are attracted to the point. On contact, electrons are transferred (from the conductor if it is negative and to it if it is positive) and hence the air molecules become ionised. Now the same charge as the conductor, they repel away, ionising other air molecules that they make contact with. This can cause charge to leak slowly (corona discharge, sometimes characterised by a faint glow) or quickly (a spark, lightning being an extreme case). The light emitted is an effect caused by the ionisation of gas molecules.

Photocopiers and Laser Printers

Some materials (such as selenium) are *photoconductive*, that is they conduct in light but not in darkness. Photocopiers and laser printers use this property and corona discharge to attach toner (powdered ink) to paper.

1. A 'corona wire' is a highly charged (thousands of volts) wire which runs along the length of a selenium coated conducting drum. As the drum rotates, corona discharge gives the surface of the drum (currently in darkness so that the charge doesn't leak away through the drum) a positive charge.
2. Light reflected from a page (in a photocopier) or emitted by a laser (in a laser printer) falls on the drum, so that it conducts more in the locations on the drum corresponding to where the page is lighter (and the charge leaks away through the drum there, leaving those spots with less or no charge).
3. The drum attracts negatively charged toner onto its charged (dark) spots.
4. Paper is passed by another corona wire which charges the back of it positively (more so than the charge on the drum).
5. The positively charged paper attracts the negative toner from the drum.
6. A corona wire fed by an alternating current sprays whichever charges are attracted to the paper, neutralising its charge. This is done not just because having charged paper coming out of a photocopier would be a problem, but also because being charged means the paper tends to 'cling' to the positive drum because of the negative toner on the drum side of the paper.
7. The paper passes between a heat and a pressure roller which melts the toner and fuses it to the paper.

Note: This is a negative toner/positive drum copier example – it would work just fine if all the charges were swapped.