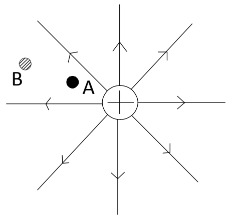
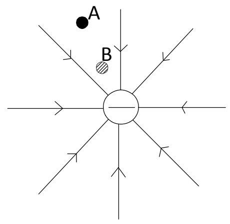
The Motion of Charged Particles in Electric Fields

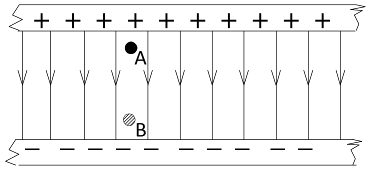
Electric Potential Difference

Any two points in an electric field have an electric potential difference between them. An analogy to this is the gravitational potential energy difference between two points in a gravitational field. If you lift an object 2m above the ground, it has more potential to fall than if it were 1m above the ground.

A positive test charge in an electric field has a certain amount of electric potential. If it moves along the field lines, it loses potential.







On the diagrams above, point A has a higher potential, and point B has a lower potential.

Electric potential difference between two points in an electric field is defined by the equation $\Delta V = \frac{W}{q}$ where

ΔV is potential difference and W is the work that would be done on a charge of q to move it between the two points.

Potential difference is given in units of volts (v), or joules per coulomb (J C^{-1}).

For small energies, sometimes the units of joules are not used, values are instead given in electron volts (eV).

1 electron volt is enough energy to move an electron through 1V.

To convert from J to eV, divide by 1.60×10^{-19}
To convert from eV to J, multiply by 1.60×10^{-19}

Given two oppositely charged parallel plates a distance d apart and the potential difference ΔV between them, we can derive an expression for the magnitude of the uniform electric field between them (I.e. not at the edges).

The energy required to move a charged particle q from one plate to the other would be $W = q\Delta V$.
We also know that the force on a charged particle anywhere in the field is $F = Eq$.
Now $W = Fd$ so we have $W = Eqd = q\Delta V$.

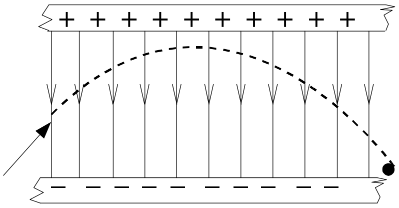
$$Eqd = q\Delta V$$
$$\therefore Ed = \Delta V$$
$$\therefore E = \frac{\Delta V}{d}$$

Motion of a Charged Particle in a Constant Electric Field

A uniform electric field will produce a constant force (magnitude and direction) on a particle with constant charge, therefore constant acceleration.

The acceleration of a charge q of mass m in a uniform electric field $\vec{E}$ is given by $\vec{a} = \frac{q\vec{E}}{m}$

Since this is a **constant** in a uniform field, the motion of a charged particle in a uniform field will be just like that of an object undergoing projectile motion – except that the acceleration will be due to the electric field present rather than gravity.



(Note: gravity is ignored in this example)

The path shown in the example above is that of a proton. Notice that it produces a parabola, just like in projectile motion.

As in projectile motion, there is no acceleration perpendicular to the field lines, so the field-perpendicular component (horizontal in this case) is constant (ignoring any kind of friction).

If the particle is travelling parallel or antiparallel to the field lines, the problem becomes one dimensional.

Given a charged particle entering a uniform field at right angles at some speed, the time of flight and the total deflection by the time it has travelled the entire length of the field (as it leaves the field) can be calculated as follows:

1. Use the initial speed (v_0) and length of field (l) to find the time t the particle spends in the field, since the particle enters with its speed being the entire field-perpendicular component of velocity and this remains constant.

   $t = \frac{l}{v_0}$ $\{$ since speed = distance travelled over time taken$\}$

2. Find the magnitude of acceleration due to the electric field, using $a = \frac{qE}{m}$

3. Find the deflection of the particle using the equation $s = v_0 t + \frac{1}{2}at^2$

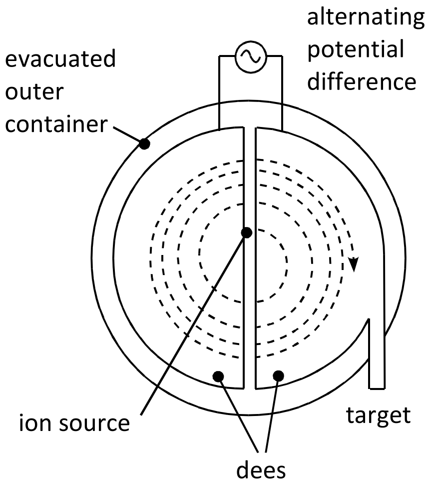
   Note: Remember to use signs consistently.

Application: The Use of Electric Fields in Cyclotrons

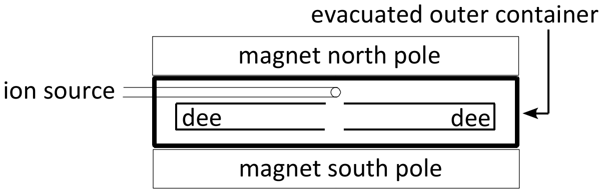
A cyclotron is a device for accelerating charged particles to high speeds, usually so that they can be collided with atomic nuclei to produce radioactive material (for example for hospital use).

A cyclotron consists of:

- An **ion source**, which gives hydrogen atoms a negative or positive charge
- Two semicircular metal containers called **dees** because they resemble the capital letter D
- An **evacuated outer container** (the shell of the cyclotron, which contains as few air molecules as possible to minimise collisions which would slow down the particles being accelerated)
- An **electromagnet** (maintains the circular motion of the particles)



Top view



Side view

There will be approximately no **electric** field inside the dees, since they are almost closed hollow conductors.

Between the dees will be an alternating uniform electric field, so the ions will experience acceleration.

A property of the magnetic field present is that it causes the radius of a particle's motion to be proportional to its speed, meaning that it will always take the same amount of time to complete an orbit. This allows the particle to always be going the right direction to match the alternating electric field (it changes every half-revolution of the particle), so that it speeds up each time.

This process is described as “accelerating particles to high energies” since every time the particle is sped up across the gap it is imparted more kinetic energy. This increase in kinetic energy each crossing of the gap is equal to the amount of work done in accelerating the particle (both work and kinetic energy are measured in joules), $\Delta K = W = q\Delta V$.

It is very important that the chamber be evacuated not just so that no collision cause the ions to lose energy; it is also important that they not be scattered away from their circular path as they could collide with the walls. If such collisions were to take place, the atoms in the wall could be modified and become radioactive which would interfere with the results.