Motion of Charged Particles in Magnetic Fields

Force on Charged Particles in Magnetic Fields

Moving electric charges in a magnetic field experience a force.

$F = qvB \sin \theta$

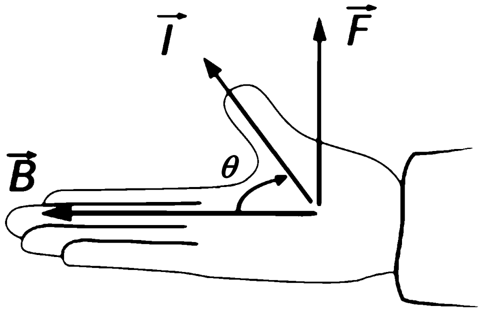
Here $v \sin \theta$ is the component of the velocity of the particle perpendicular to the field.

Notice that the magnetic force is velocity-dependent (the slower the particle moves, the less force it experiences).

Also important to notice is that the force is

- strongest when the velocity is perpendicular (90° or 270°) to the field, since sin(90°) = 1 and sin(270°) = 1
- zero when the velocity is parallel or antiparallel (0° or 180°) to the field, since sin(0°) = 0 and sin(180°) = 0

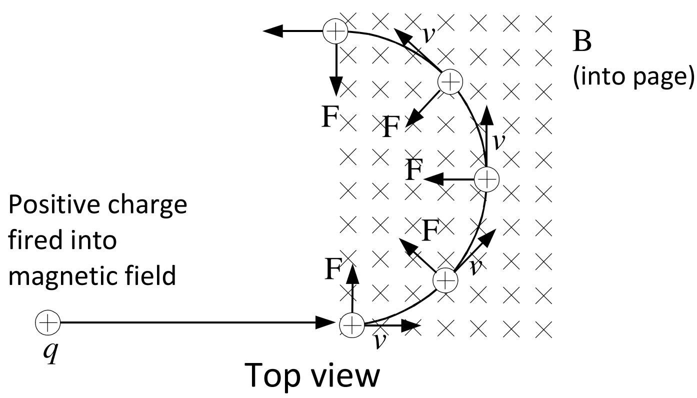
The direction of the magnetic force is at right angles to the plane defined by the directions of the velocity and magnetic field and is given by the right-hand rule below:

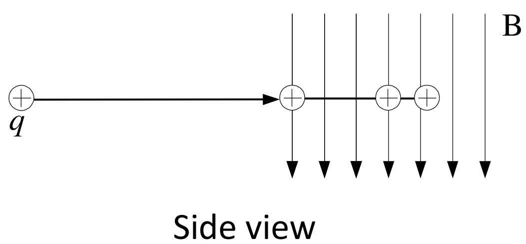


Notice that directions match that of current flowing in a magnetic field – the direction of 'positive flow', by convention.

Motion at Right Angles: Uniform Circular Motion

Charged particles moving parallel or antiparallel to the magnetic field experience no force. Any angle between these and perpendicular experiences complex motion and will not be considered.
A charged particle moving at right angles to a uniform magnetic field experiences a force of constant magnitude at right angles to the velocity, and hence moves with uniform circular motion. For example:





A negative particle would travel in the opposite direction, but still in uniform circular motion.

This uniform circular motion is a result of the velocity-dependence of the magnetic force. As the force acts, the velocity changes (the direction, not the speed) which changes the direction of the force.

The velocity stays at right angles to the magnetic field, as seen in the side view diagram.

Since the particle is travelling with uniform circular motion, an equation can be derived for the radius of the curve for some particle of mass *m.*

$$a = \frac{v^2}{r} \quad \{\text{centripetal acceleration}\}$$

$$\therefore F = \frac{mv^2}{r} \quad \{a \text{ inserted into } F = ma\}$$

$$F = qvB\sin\theta$$

$$= qvB \quad \{\text{since } \theta = 90° \text{ so } \sin\theta = 1\}$$

$$\therefore qvB = \frac{mv^2}{r} \quad \{\text{equating the two equations for } F\}$$

$$\therefore r = \frac{mv}{qB}$$

The Use of Magnetic Fields in Cyclotrons

For details on the construction of the cyclotron (and diagrams of the cyclotron), see the notes on the motion of charged particles in electric fields.

The magnet above and below the dees of the cyclotron maintain the circular path of a charged particle (ion).

The magnetic field is set up by having one magnetic pole above the cyclotron and the other below. The field created is uniform and down into the cyclotron or up through the cyclotron, depending on the direction and charge of the particle.

An expression for the period of motion of the ion around the cyclotron can be derived:

T is the period (time taken to orbit once)

$$C = 2\pi r \quad \text{and} \quad \text{speed} = \frac{\text{distance}}{\text{time}} \quad \therefore v = \frac{2\pi r}{T}$$

$$r = \frac{mv}{qB}$$

$$\therefore r = \frac{m\frac{2\pi r}{T}}{qB} \quad \{\text{inserting the equation for } v\}$$

$$\therefore T = \frac{2\pi m}{qB}$$

The *v* and *r* terms do not appear anywhere in the expression; the period of motion is *independent* of the speed and radius of the ion. This is key to the operation of the cyclotron. The particle's speed and radius increase but the period of motion is constant, so the electric field between the dees can alternate at a constant rate, and the particle will always be crossing the field at the right time to gain energy.

When the ions reach some outer radius *r*, they exit the cyclotron with some kinetic energy *K:*

$$K = \tfrac{1}{2}mv^2$$

$$r = \frac{mv}{qB} \quad \therefore v = \frac{qBr}{m}$$

$$\therefore K = \tfrac{1}{2}m\left(\frac{qBr}{m}\right)^2 \quad \{\text{inserting } v\}$$

$$= \frac{q^2B^2r^2}{2m}$$

Notice that *K* is therefore independent of the potential difference or electric field between the dees. If the ions' charge is a constant, *K* depends only on the magnetic field strength and the radius of the cyclotron.