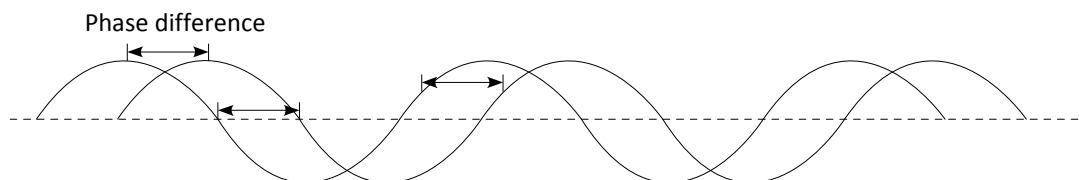# The Interference of Light

## Coherent Wave Sources

Wave sources are *coherent* if they maintain a constant phase relationship with each other. That is, the *phase difference* between their oscillation remains constant over time.



Phase difference

This diagram shows  an example of waves which have a constant phase difference.

In order for the phase difference to remain constant, the *sources* must have the same frequency.
This also means that for light to be coherent, it must be *monochromatic* (a single frequency). This can be seen from the fact that the word monochromatic means "single colour" and colour is seen according to the frequency of light waves.

Two coherent wave sources are *in phase* if their phase difference is zero (they are creating the same part of the wave at the same time), and *out of phase* otherwise. The phase difference can be given in terms of wavelength or as an angle where $\pi$ radians (180°) is half a wavelength out of phase (one source is creating a crest while the other is creating a trough).

An incandescent light (a filament that glows due to heat) is an example of an *incoherent* light wave source (as opposed to a *coherent* wave source). The electrons are, due to heat, oscillating at a range of frequencies and therefore producing light at a range of frequencies (so the light is not monochromatic). Since only monochromatic light can be coherent, incandescent bulbs produce incoherent light. Even if the light were monochromatic it would still not be coherent since the electrons are not oscillating in phase with each other at all times.

## Interference

Whenever two or more electromagnetic waves overlap, their electric and magnetic fields follow the *principle of superposition* and produce a resultant wave which is a vector addition of the electric and magnetic field vectors of each wave at that point.
When the waves at a point are *in phase* the resultant amplitude is the sum of the individual amplitudes. This is referred to as 'constructive interference' or 'reinforcement'.
When the waves at a point are *out of phase* the resultant amplitude is the difference between the individual amplitudes. This is referred to as 'destructive interference' or 'annulment'.

## Two-source Interference

If there are two monochromatic, coherent, in phase sources, their light will experience interference. The result is called an *interference pattern*.
The distance to any point in the interference pattern can be expressed  as a *path difference* – the difference in the distances from the point to each source. Path difference is often given in terms of wavelengths ($\lambda$) since both sources produce the same wavelength of light.

The behaviour of intersecting waves form these two sources  can be predicted, they:
• constructively interfere when the path difference from the sources to the point is $m\lambda$
• destructively interfere when the path difference from the sources to the point is $(m + \frac{1}{2})\lambda$
  where $m$ is an integer

# Diffraction

Whenever a wave passes an obstacle it will spread out afterwards; this effect is known as *diffraction.* For slits, the spreading will be greater when the slit size is similar to the wavelength.

Diffracted waves
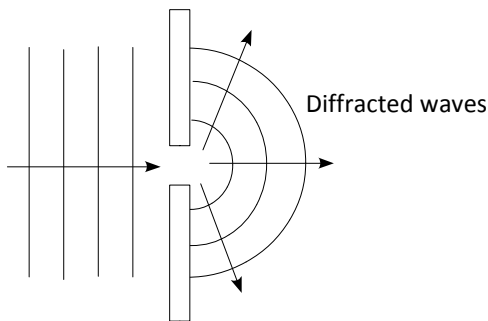
# Two-slit Interference

Two coherent and in phase light sources are difficult to create, but the same effect can be produced by passing coherent, monochromatic light through two thin slits. Diffraction causes the slits to act like new sources of light which are coherent and in phase, so the interference patterns can be observed.

Often a laser is used as the initial source as it produces coherent, monochromatic light. Any monochromatic light source can be used, but if it is not coherent then it needs to pass through a single slit before it reaches the two slits. The light passing through the single slit is diffracted and and acts like a coherent light source.

It is important that the light reaching the two slits be coherent and reach them at the same time (so the two slits must be the same distance from the source) so that the light the slits produce is in phase (and therefore will exhibit the desired interference patterns).

Monochromatic source

$d$

Coherent waves

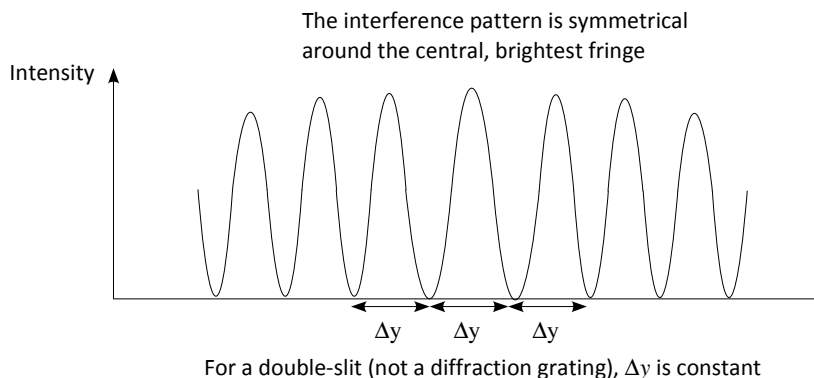Bright and dark fringes

Pattern on screen

$y$
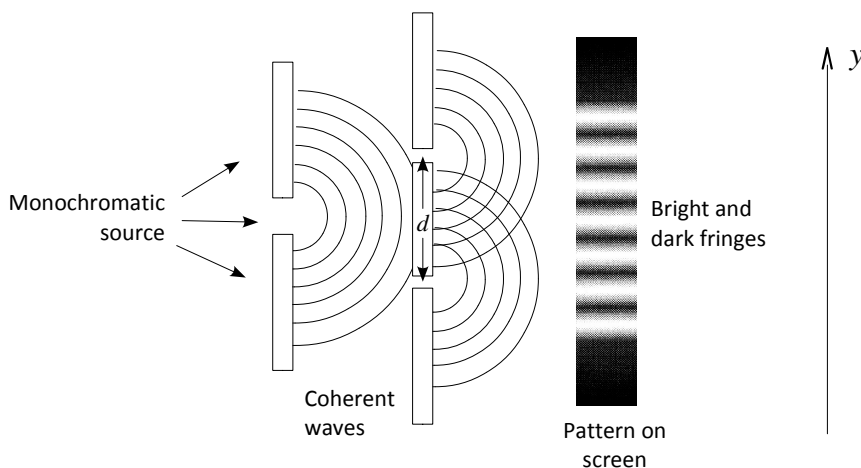
Notice that it is the diffraction (spreading out) of light in this two-slit apparatus that allows it to overlap and therefore interfere.

Each bright area is a 'bright fringe' and each dark area is a 'dark fringe'. The distance between fringes (distance from bright to bright or dark to dark) is called the fringe width, denoted $\Delta y$ and is constant.

The bright fringes are the product of constructive interference and the dark fringes are the product of destructive interference.

When discussing the maxima, they can be referred to by how many maxima they are away from the central maximum. The central maximum is the $0^{th}$ maximum, the ones to the left and right of it are the $1^{st}$ maxima, the next ones out are the $2^{nd}$ maxima, and so on. So any maximum can be described as the $m$th order maximum, where $m$ is an integer.

A graph of intensity of the pattern would look like this:

The interference pattern is symmetrical
around the central, brightest fringe



For a double-slit (not a diffraction grating), $\Delta y$ is constant

If it is assumed that the distance $L$ from slits to the screen is much greater than the distance between slits $d$, an equation which describes the angular position $\theta$ of the $m$th maximum for any wavelength $\lambda$ and distance between slits $d$ can be derived as follows:
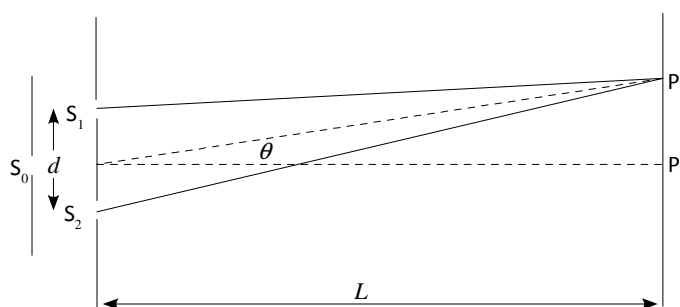


Keep in mind the diagram above is very much out of scale, since $L$ should be *much* greater than $d$ (for example $d$ might be $1 \times 10^{-6}$ m and $L$ might be 1 m).  A diagram of the two slits *in scale* would look more like this:

Zoomed in on the two slits:                                                    Zoomed out to show slits and screen:
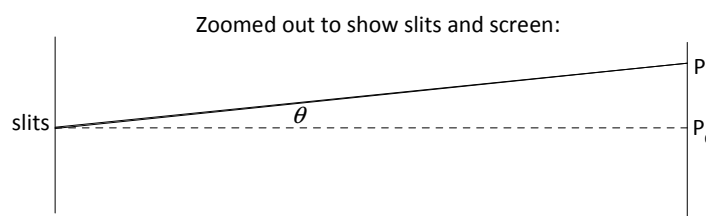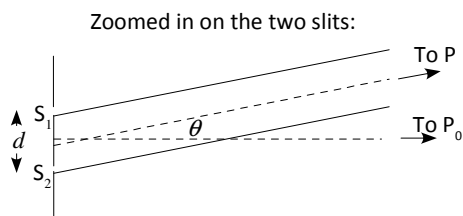


The large distance from the slits to P causes $S_1P$ and $S_2P$ to be approximately parallel. $S_2P$ is slightly longer than $S_1P$, and their difference in length (*path difference*) can be determined from a right-angled triangle drawn as shown below.
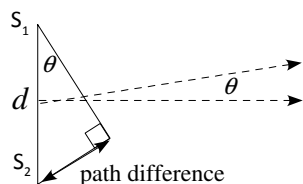


The dashed lines to P and $P_0$ have been included here to help show how this diagram was drawn from the above diagrams. Including them is necessary if you need to show why the angle at the top of the triangle is equal to $\theta$.

According to trigonometry:

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\therefore \sin \theta = \frac{\text{path difference}}{d}$$

Maxima (bright fringes) occur when the path difference is $m\lambda$, and minima (dark fringes) occur when the path difference is $(m+\frac{1}{2})\lambda$ , where $m$ is an integer (whole number) called the *order* of the fringe, for example the central maximum is the $0^{\text{th}}$ order ($m = 0$), the maxima either side of the centre are the $1^{\text{st}}$ order maxima (m = 1), etc.

From the derivations above, we can conclude $d \sin \theta = m\lambda$ for maxima and $d \sin \theta = (m+\frac{1}{2})\lambda$ for minima.

The relationship between $\theta$, $\Delta y$ and $L$ can be determined from the diagram below:
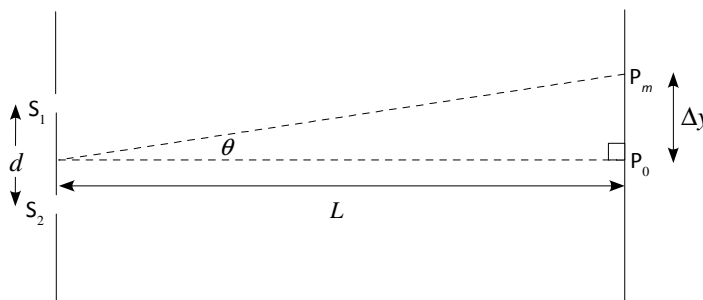


$$\tan \theta = \frac{\Delta y}{L} \quad \left\{ \tan \theta = \frac{\text{opposite}}{\text{adjacent}} \right\}$$

If $\lambda$ is unknown, it can be calculated by determining $\theta$ (using the above formula) and then substituting into $d\sin\theta = m\lambda$

If $\theta$ is small, then $\tan\theta \approx \theta$, so a relationship can be derived between $\theta$, $\lambda$, $\Delta y$ and $L$:

$$d \sin \theta = \lambda$$

$$\therefore d \tan \theta = \lambda \quad \{\sin \theta \approx \tan \theta\}$$

$$\therefore d \frac{\Delta y}{L} = \lambda \quad \left\{ \tan \theta = \frac{\Delta y}{L} \right\}$$
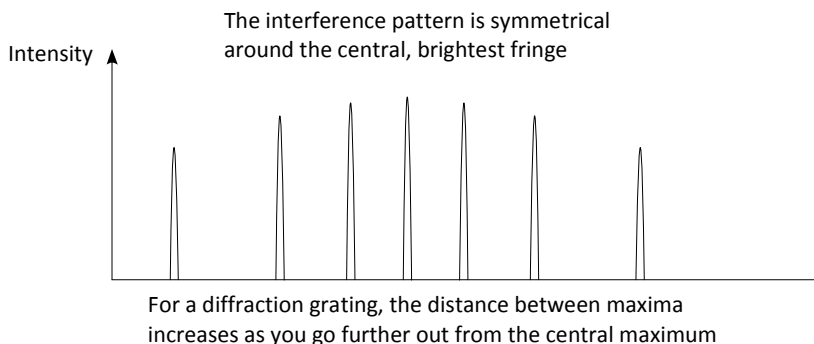
$$\therefore \Delta y = \frac{\lambda L}{d}$$

Remember: this is only accurate for very small values of $\theta$. A reasonable guideline would be to only use it if $\theta$ is less than 1°.


## Transmission Diffraction Gratings

A transmission diffraction grating consists of many very thin, equally spaced parallel slits. These slits are usually created by scratching a transparent surface – light will reflect back from the scratched sections but pass through the unscratched sections as if they are slits.

Often a grating is stated as having a number of slits per metre. This number is denoted $N$ and can be converted to or from distance between slits using the equation $d = 1/N$

The result of the large number of slits is that the diffraction of light from each slit allows it to overlap the other light and so the interference pattern consists of very narrow intense maxima separated by regions of negligible intensity. Due to diffraction and the high number (many hundreds) of slits, the pattern can even feature intense maxima on large angles all the way up to 90°. As for two-slit interference, intensity decreases with larger angles:



The interference pattern is symmetrical around the central, brightest fringe

For a diffraction grating, the distance between maxima increases as you go further out from the central maximum

The equation for angles of intensity maxima in this pattern can be derived in a similar fashion to the derivation for two slits, the most important difference being that the initial diagram will show construction lines to P from more than two slits, something similar to that shown to the right.
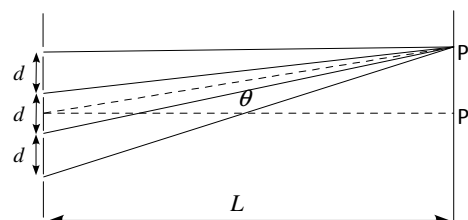
Since the angle of diffraction cannot ever be greater than 90°, the maximum number of orders (the total number of orders is the number of maxima on *one* side of the central maximum) possible for any given wavelength passing through a slit with a given distant between slits.

$$\sin\theta = \frac{m\lambda}{d}$$

Insert the values for $\lambda$ and $d$. $\theta > 90$ when $\sin\theta > 1$, so the maximum value for $m$

is the value *before which* $\dfrac{m\lambda}{d} > 1$

If the number of slits per distance is given instead of the distance between slits, simply convert between using the equation $d = 1/N$

The pattern caused by the two-slit interference involves a smooth transition from maxima to minima, whereas the pattern caused by a diffraction grating has very small, sharp peaks separated by large areas of destructive interference. The reason for the large regions of complete destructive interference is the high number of slits. While the waves from nearby slits may be only slightly out of phase, light from many hundreds of slits away may be exactly half a wavelength out of phase and therefore completely annul. The more slits there are, the more complete this destructive interference can be.

The wavelength of any monochromatic light source can be calculated by passing it through a diffraction grating with a known distance between slits, and measuring the angles of the maxima.

If white light is shone through a diffraction grating, an example of the resulting interference pattern is as shown below (only one side is shown):
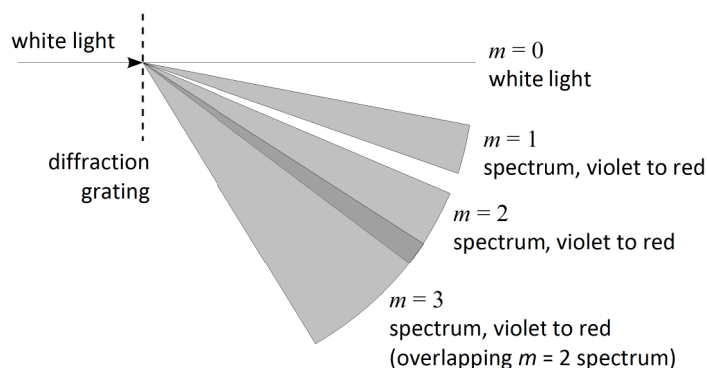


- central maximum is white because light of every colour reinforces there (dsinθ = 0)
- next to the central maximum is a dark area because light of all wavelengths will destructively interfere there
- next is a continuous spectrum, starting at violet because it has the smallest wavelength therefore smallest θ for constructive interference
- next is another area of annulment followed by another spectrum and so on. These spectra will be fainter and fainter and start to overlap (no area of annulment) by the third order

Diffraction gratings are useful for a process called *spectroscopy* (study of electromagnetic spectra) for the following reasons:
- for close slits, the angular deflection is very large and therefore the angle of any given wavelength can be measured with low error (high precision). The spectrum is clearer and more dispersed than for a triangular prism
- diffraction gratings can spread out the wavelengths of light, allowing for identification of wavelengths for a source that is not monochromatic (this allows for identification of individual atoms and elements since each produce their own spectral lines)
- the grating formula $d\sin\theta = m\lambda$ makes it easy to calculate the wavelengths

## Speckle

The light produced by a laser is parallel, monochromatic and coherent. When it reflects off a rough surface (variations in level greater than ¼λ) it goes in various directions and arrives at our eyes at different points. The resulting path differences cause interference – bright spots of constructive interference and dark regions of destructive interference.

## Compact Discs and DVDs

Information on a compact disc (CD) or digital versatile disc (DVD) is stored as a continuous spiral of ¼ λ deep *pits* (seen as *bumps* on the laser side) around the disc. *Coherent* light is reflected from the disc and detected by a photodiode*.*

While the beam is over the *land* (the flat surface) the beam experiences constructive interference as there is no path difference.

When the beam is over a bump (pit) the light returning from the edge of the bump is out of phase with the light returning from the land, and destructive interference occurs.
The intensity of light detected by the photodiode therefore changes as the beam passes from bump to land or land to bump, allowing the data to be read.

← direction of motion of disc ←
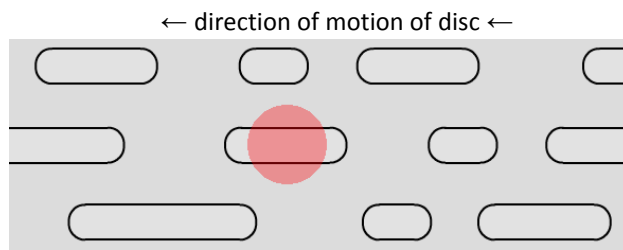


The beam in this diagram is currently on a bump.

Since the CD is constantly spinning and the laser needs to scan around the disc to read different parts of the disc, a diffraction grating is used to produce three beams from one (this method of keeping the laser on the correct track of a CD is called the three-beam method). The central (highest intensity) beam is used to read the data, while the other two beams detect whether the beam is on track.
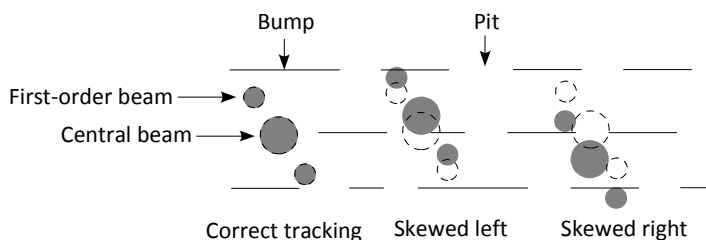


If the beam is off track, the first-order beam in that direction will experience destructive interference whenever it passes over a bump, so changes in intensity will be detected. The beam can then be shifted back into position.
A DVD is able to store more data than a CD by using a shorter wavelength laser (650 nm rather than 780 nm). The shorter the wavelength, the higher the number of pits and bumps that can be detectable on a disc. Blu-ray discs use a wavelength that is shorter again (405 nm).