## **SOLUTIONS**

## Projectile Motion Test 2

1.

(a) 
$$v_H = v_0 \cos \theta = 12.3 \times \cos 26.0^\circ = 11.1 \text{ ms}^{-1}$$
  
 $v_V = v_0 \sin \theta = 12.3 \times \sin 26.0^\circ = 5.39 \text{ ms}^{-1}$ 

(b) 
$$s_V = v_{0V}t + \frac{1}{2}a_Vt^2$$

Lands at launch height  $\therefore s_v = 0$ 

$$0 = v_{0} t + \frac{1}{2} a_V t^2$$

$$\therefore 0 = t \left( v_{0_V} + \frac{1}{2} a_V t \right)$$

: 
$$t = 0$$
 or  $v_{0_V} + \frac{1}{2}a_V t = 0$ 

$$\therefore t = \frac{-v_{0_V}}{\frac{1}{2}a_V} = \frac{-5.39}{\frac{1}{2} \times -9.8} = 1.10 \text{ s}$$

(c) 
$$s_H = v_H t = 11.1 \times 1.10 = 12.2 \text{ m}$$

(d) 
$$v_V^2 = v_{0_V}^2 + 2a_V s_V$$

At max height,  $v_v = 0$ 

$$\therefore 0 = v_{0_V}^2 + 2a_V s_V$$

$$\therefore -v_{0_V}^2 = 2a_V s_V$$

$$\therefore s_V = \frac{-v_{0_V}^2}{2a_V} = \frac{-5.39^2}{2 \times -9.8} = 1.48 \text{ m}$$

(e) 
$$v_V = v_{0_V} + a_V t$$
  
= 5.39 + -9.8 × 0.87  
= -3.1 ms<sup>-1</sup>

$$v_H = 11.1 \text{ ms}^{-1}$$

$$\theta$$

$$v_V = 3.1 \text{ ms}^{-1}$$

$$\theta = \tan^{-1} \left(\frac{v_V}{v_H}\right) = \tan^{-1} \left(\frac{3.1}{11.1}\right) = 16^{\circ}$$

$$v = \sqrt{v_H^2 + v_V^2} = \sqrt{11.1^2 + 5.39^2} = 12 \text{ ms}^{-1}$$

The final velocity of the soccer ball is 12 ms<sup>-1</sup> at 16° below the horizontal.

(f) Decreasing the angle decreases the vertical component of initial velocity, which decreases the time of flight. The horizontal component is slightly increased but not enough to counteract this change.

2. Air resistance is a force which opposes motion, slowing the velocity of the shuttlecock. In this case therefore there will be a force acting upwards on the projectile, reducing its downward acceleration and therefore increasing the time of flight.

3.

(a) 
$$v_V = v_{0_V} + a_V t$$
  
 $v_{0_V} = 0$  when dropped  
 $\therefore v_V = a_V t$ 

$$v_{f} = \sqrt{v_{H}^{2} + v_{V}^{2}}$$

$$= \sqrt{v_{H}^{2} + (a_{V}t)^{2}}$$

$$\therefore v_{f}^{2} = v_{H}^{2} + (a_{V}t)^{2}$$

$$\therefore (a_{V}t)^{2} = v_{f}^{2} - v_{H}^{2}$$

$$\therefore g^{2}t^{2} = v_{f}^{2} - v_{H}^{2}$$

$$\therefore t^{2} = \frac{v_{f}^{2} - v_{H}^{2}}{g^{2}}$$

$$\therefore t = \sqrt{\frac{v_{f}^{2} - v_{H}^{2}}{g^{2}}}$$

(b) There are multiple approaches to this question. In this example, we calculate the time of flight and the time to reach 200 ms<sup>-1</sup>, and compare.

Time of flight:

$$s_V = v_{0_V} t + \frac{1}{2} a_V t^2$$

 $v_{0_v} = 0$  when dropped

$$\therefore s_V = \frac{1}{2} a_V t^2$$

$$\therefore t = \sqrt{\frac{s_V}{\frac{1}{2}a_V}} = \sqrt{\frac{-2.2 \times 10^3}{\frac{1}{2} \times -9.8}} = 21.2 \text{ s}$$

Time to reach 200 ms<sup>-1</sup>:

$$t = \sqrt{\frac{{v_f}^2 - {v_H}^2}{g^2}} = \sqrt{\frac{200^2 - 68^2}{9.8^2}} = 19.2 \text{ s}$$

The GoPro will reach a speed of 200 ms<sup>-1</sup>.