

Gravitation and Satellites (+ Banking Angle)

1.

$$(a) T = \frac{2\pi r}{v}$$

$$\therefore r = \frac{Tv}{2\pi}$$

$$v = \sqrt{\frac{GM}{r}}$$

$$\therefore r = \frac{T\sqrt{\frac{GM}{r}}}{2\pi}$$

$$\therefore r^2 = \frac{T^2 \frac{GM}{r}}{4\pi^2} = \frac{T^2 GM}{4\pi^2 r}$$

$$\therefore r^3 = \frac{T^2 GM}{4\pi^2}$$

$$\therefore r = \sqrt[3]{\frac{T^2 GM}{4\pi^2}}$$

(b) Geostationary orbit means a period of 24 hours, or 86400 seconds.

$$\therefore r = \sqrt[3]{\frac{86400^2 \times 6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{4\pi^2}} = 4.23 \times 10^7 \text{ m}$$

$$(c) v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{4.22 \times 10^7}} = 3.07 \times 10^3 \text{ ms}^{-1}$$

2.

(a) Low altitude polar, as they are able to see any point on the Earth undistorted and close up.

(b) Low-altitude satellites require high orbital speeds. Launching into equatorial orbit in the direction of Earth's rotation (West-to-East) provides some of this speed on launch.

3.

(a) Gravitational force is providing the centripetal acceleration, and this force is towards the centre of mass of the planet.

$$(b) F = G \frac{m_1 m_2}{r^2}$$

 G, m_1 and r are constant

$$\therefore F \propto m_2$$

Let the Endor/Forest Moon system be A, and Earth/Moon be B.

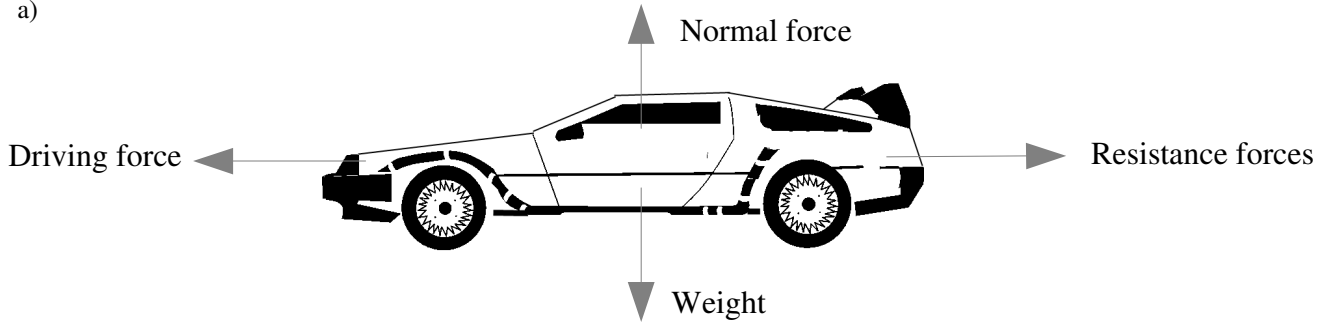
$$\therefore \frac{F_A}{F_B} = \frac{m_{2A}}{m_{2B}} = \frac{4m_{2B}}{m_{2B}} = 4$$

$$\therefore F_A = 4F_B = 4 \times 2.03 \times 10^{20} = 8.12 \times 10^{20} \text{ N}$$

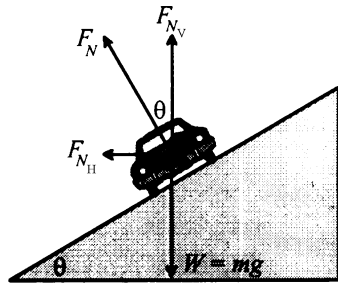
(c) Newton's third law states that every action has an equal and opposite reaction. Endor and its moon attract each other with equal force, but in opposite directions (one towards Endor, one towards the Forest Moon).

4.

a)



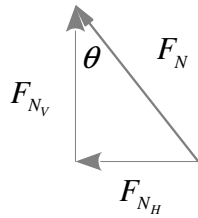
b) Banking a curve means that the normal force (the road on the car) has a horizontal component. This horizontal component provides some (or all) of the centripetal acceleration for a car taking the curve. This means the friction does not need to provide as much acceleration.



In the diagram, F_{NH} can be seen to provide at least some of the centripetal acceleration. /4

c) The vertical component must still be sufficient to keep the car from sinking into the road, so $F_{Nv} = mg$. For the horizontal component to provide exactly all the centripetal acceleration (friction of the tyres not needed) $F_{Nh} = F_c = ma_c = m \frac{v^2}{r}$

The total normal force is the vector sum of its components, so:



$$\tan \theta = \frac{F_{Nh}}{F_{Nv}}$$

$$\therefore \tan \theta = \frac{m \frac{v^2}{r}}{mg}$$

$$\therefore \tan \theta = \frac{v^2}{rg}$$