Proportionality Practice Questions

1. Consider two points A and B in the electric field produced around a point charge. The points are at radius r and $4r$ from the charge, respectively. Using proportionality, calculate the magnitude of electric field strength at point B, given that the electric field strength at point A is 20 NC^{-1}.

2. Two moons X and Y are orbiting a planet in such a way that they are experiencing equal magnitude of force of gravitational attraction from the planet. The moons' masses are in ratio $m_X : m_Y = 1 : 2$. Calculate, using proportionality, the ratio of the radii of the moons around the planet.

3. Two satellites C and D are orbiting a planet at radii of r and $4r$ respectively. Calculate the ratio $v_C : v_D$ of the satellites' orbital speeds, and $T_C : T_D$ of their orbital periods.

Proportionality Practice ***<u>Solutions</u>***

1. $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad \therefore E \propto \frac{1}{r^2} \quad$ since $\frac{1}{4\pi\varepsilon_0}$ and q are constant

$$\therefore \frac{E_1}{1/r_1^2} = \frac{E_2}{1/r_2^2}$$

$$\therefore E_2 = \frac{E_1/r_2^2}{1/r_1^2} = \frac{E_1 r_1^2}{r_2^2} = \frac{20r^2}{(4r)^2} = \frac{20}{16} = 1.25\ \text{NC}^{-1}$$

2. Let m represent a moon mass and M represent the planet's mass

$\therefore$ for each moon, $F = G\frac{Mm}{r^2}$

$$\therefore Fr^2 = GMm$$

$$\therefore r = \sqrt{\frac{GMm}{F}}$$

$\therefore r \propto \sqrt{m} \quad$ since G, M and F are constant

$$\therefore \frac{r_X}{r_Y} = \frac{\sqrt{m_X}}{\sqrt{m_y}} = \frac{\sqrt{m}}{\sqrt{2m}} = \frac{1}{\sqrt{2}}$$

3. $v = \sqrt{\frac{GM}{r}}$

$\therefore v \propto \frac{1}{\sqrt{r}} \quad$ since G and M are constant

$$\therefore \frac{v_C}{v_D} = \frac{1/\sqrt{r_C}}{1/\sqrt{r_D}} = \frac{\sqrt{r_D}}{\sqrt{r_C}} = \frac{\sqrt{4r}}{\sqrt{r}} = \sqrt{4} = 2$$

$$\therefore v_C : v_D = 2:1$$

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{\frac{GM}{r}}} = \frac{2\pi r\sqrt{r}}{\sqrt{GM}} = \sqrt{\frac{4\pi^2 r^3}{GM}}$$

$\therefore T \propto \sqrt{r^3} \quad$ since G and M are constant

$$\therefore \frac{T_C}{T_D} = \frac{\sqrt{r_C^3}}{\sqrt{r_D^3}} = \frac{\sqrt{r^3}}{\sqrt{(4r)^3}} = \frac{1}{\sqrt{4^3}} = \frac{1}{8}$$

$$\therefore T_C : T_D = 1:8$$