

Question 3 (6 marks)

Let $A = \begin{bmatrix} \frac{1}{9} & 0 \\ 0 & 2 \end{bmatrix}$.

(a) Use mathematical induction to prove that $A^n = \begin{bmatrix} \left(\frac{1}{3}\right)^{2n} & 0 \\ 0 & 2^n \end{bmatrix}$ for all positive integers n .

(5 marks)

(b) Using part (a), find the positive integer n such that $A^n \begin{bmatrix} 0 \\ 8 \end{bmatrix} = \begin{bmatrix} 0 \\ 2^{2021} \end{bmatrix}$.

(1 mark)