Question 8 (7 marks)

(a) Prove by mathematical induction that $10^n - 6^n$ is divisible by 4 for all positive integers n.

P_n is : $4 \mid 10^n - 6^n$ for all $n \in \mathbb{Z}^+$

If $n=1$, P_1 is : $4 \mid 10 - 6$

ie. $4 \mid 4$ $\therefore P_1$ is true

If P_k is true, then P_k is: $10^k - 6^k = 4A$ for some $A \in \mathbb{Z}$

i.e. $10^k = 4A + 6^k$

P_{k+1} should be : $10^{k+1} - 6^{k+1} = 4B$ for some $B \in \mathbb{Z}$

Now $10^{k+1} - 6^{k+1} = 10 \times 10^k - 6 \times 6^k$

$= 10(4A + 6^k) - 6 \times 6^k$ using P_k

$= 40A + 10 \times 6^k - 6 \times 6^k$

$= 40A + 4 \times 6^k$

$= 4(10A - 6^k)$

$= 4B$ where $B \in \mathbb{Z}$ since $A \in \mathbb{Z}$ and $k \in \mathbb{Z}^+$

$\therefore P_{k+1}$ is true whenever P_k is true

Since P_1 is true, and P_{k+1} is true whenever P_k is true, P_n is true for all $n \in \mathbb{Z}^+$ by the principle of mathematical induction.

(5 marks)

(b) Hence find the last two digits of $25\left(10^{2018}-6^{2018}\right)$.

$4 \mid 10^{2018} - 6^{2018}$ using P_{2018}

$\therefore\ 4\times 25 \mid 25\left(10^{2018}-6^{2018}\right)$

ie. $100 \mid 25\left(10^{2018}-6^{2018}\right)$

$\therefore$ The last two digits are both zero.

(2 marks)