**Stage 2 Specialist Mathematics**

**Mathematical Induction Test**

**Topic 1: Subtopics 1.1**

**Total Marks – 25**

**(Calculator permitted. One A4 page of handwritten notes not permitted.)**

1. (7 marks)
2. Use the table features on your graphics calculator to complete the following:

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(1 mark)

1. Use your results from (a) to develop a conjecture.

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(1 mark)

***Question 1 continues on page 2.***

1. Use mathematical induction to prove your conjecture.

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(5 marks)

1. (5 marks)

Use mathematical induction to prove that, for any positive integer, :

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(5 marks)

1. (7 marks)
2. Use the matrix features on your graphics calculator to complete the following:

(1 mark)

1. Use your results from (a) to develop a conjecture.

for all

(1 mark)

***Question 3 continues on page 5.***

1. Use mathematical induction to prove your conjecture.

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(5 marks)

1. (6 marks)

Use mathematical induction to prove that, for any positive integer, :

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(6 marks)