**Stage 2 Specialist Mathematics**

**Mathematical Induction Test**

**Topic 1: Subtopics 1.1**

**Total Marks – 25**

**(Calculator permitted. One A4 page of handwritten notes not permitted.)**

1. (7 marks)
2. Use the table features on your graphics calculator to complete the following:

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| $$n$$ | $$3^{2n+4}-2^{2n}$$ |
| 1 |  |
| 2 | 6545 |
| 3 |  |
| 4 |  |
| 5 | 4781945 |

(1 mark)

1. Use your results from (a) to develop a conjecture.

|  |  |  |
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| $3^{2n+4}-2^{2n}$ is divisible by |  | for all $n\in Z^{+}$ |

(1 mark)

***Question 1 continues on page 2.***

1. Use mathematical induction to prove your conjecture.

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(5 marks)

1. (5 marks)

Use mathematical induction to prove that, for any positive integer, $n$:

$$1×2+2×3+3×4+\cdots +n\left(n+1\right)=\frac{n\left(n+1\right)\left(n+2\right)}{3}$$

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(5 marks)

1. (7 marks)
2. Use the matrix features on your graphics calculator to complete the following:
3. $\left[\begin{matrix}1&1\\0&1\end{matrix}\right]^{2}=\left[\begin{matrix}&\\&\end{matrix}\right]$
4. $\left[\begin{matrix}1&1\\0&1\end{matrix}\right]^{3}=\left[\begin{matrix}&\\&\end{matrix}\right]$
5. $\left[\begin{matrix}1&1\\0&1\end{matrix}\right]^{4}=\left[\begin{matrix}&\\&\end{matrix}\right]$

(1 mark)

1. Use your results from (a) to develop a conjecture.

$\left[\begin{matrix}1&1\\0&1\end{matrix}\right]^{n}=\left[\begin{matrix}&\\&\end{matrix}\right]$ for all $n\in Z^{+}$

(1 mark)

***Question 3 continues on page 5.***

1. Use mathematical induction to prove your conjecture.

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(5 marks)

1. (6 marks)

Use mathematical induction to prove that, for any positive integer, $n$:

$$\cos(\left(x\right))×\cos(\left(2x\right))×\cos(\left(4x\right))×\cdots ×\cos(\left(2^{n-1}x\right))=\frac{\sin(\left(2^{n}x\right))}{2^{n}\sin(x)}$$

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(6 marks)