Question 10 (15 marks)

(a) (i) On the Argand diagram in Figure 8:

- mark and label a point to represent a complex number z
- hence mark and label the point representing $z+2$.

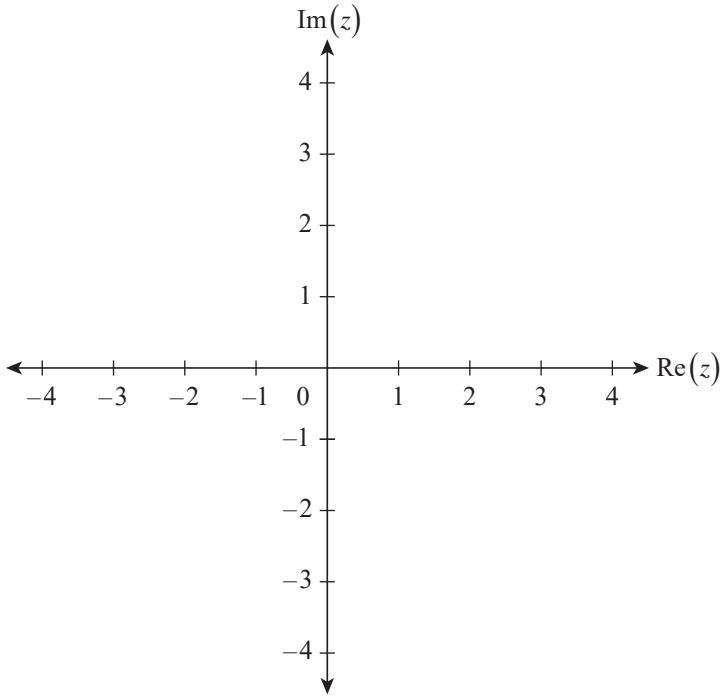

Figure 8

(1 mark)

(ii) Explain why $|z|+|z+2|\geq 2$.

(2 marks)

(iii) Explain why the only solutions for $|z|+|z+2|=2$ are real.

(1 mark)

(b) (i) On the Argand diagram in Figure 9:

(1) draw the set of all complex numbers z such that $|z+2|=|z|$. (2 marks)

(2) mark a point P, representing a complex number z such that $|z+2|=|z|$ and $\text{Im}(z)>0$. (1 mark)

(3) mark the point Q, representing $z+2$. (1 mark)

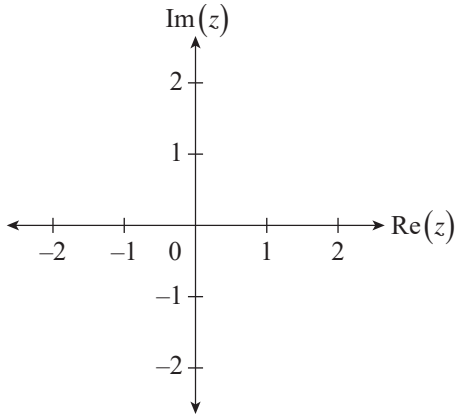

Figure 9

(ii) Let $\angle POQ=\theta$.

Show that $\dfrac{z}{z+2}=\text{cis}\,\theta$.

(2 marks)

Question 10 continues on page 10.

(c) (i) Using z from part (b), and $\theta = \frac{\pi}{6}$, write $\frac{z}{z+2}$ in Cartesian form.

(1 mark)

(ii) Hence show that $z = -1 + \left(2 + \sqrt{3}\right)i$.

(2 marks)

(iii) Using Figure 9 or otherwise, show that z may be written in polar form as $z = \operatorname{cosec} \frac{\pi}{12} \operatorname{cis} \frac{7\pi}{12}$.

(2 marks)