QUESTION 13 (15 marks)

(a) (i) Solve $z^5 = -1$. Write your solutions in polar form.

(3 marks)

(ii) Draw the solutions on the Argand diagram in Figure 14, labelling each solution in an anticlockwise direction from z_1 to z_5, where z_1 is the solution with the smallest positive argument.

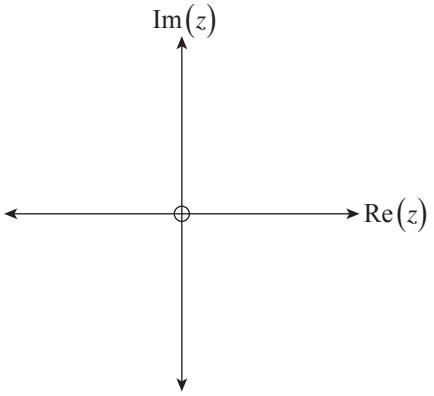

Figure 14

(2 marks)

Join your solutions labelled z_1, z_2, z_3, z_4, and z_5 to form a pentagon.

(iii) Show that $|z_1 - z_5| = 2\sin\frac{\pi}{5}$.

(2 marks)

(iv) Show that the perimeter of the pentagon is $10\sin\frac{\pi}{5}$.

(1 mark)

(v) Show that the area of the pentagon is $\frac{5}{2}\sin\frac{2\pi}{5}$.

(2 marks)

Consider the solutions to $z^n = -1$ for integers $n \geq 3$.

(b) A polygon is obtained by plotting and joining the solutions of $z^n = -1$.

Let $P(n)$ be the perimeter of this polygon.

(i) Write down an expression for $P(n)$.

(1 mark)

(ii) State the shape of the polygon formed as $n \to \infty$.

(1 mark)

(iii) What exact value does $P(n)$ approach as $n \to \infty$?

(1 mark)

(c) Let $A(n)$ be the area of the polygon described in part (b).

(i) Show that $A(n) = \frac{n}{2}\sin\frac{2\pi}{n}$.

(1 mark)

(ii) What exact value does $A(n)$ approach as $n \to \infty$?

(1 mark)