Question 9 (11 marks)

Let $f(x) = \sqrt{8 - x^3}$ for $x < 2$.

(a) Show that $f'(x) = \dfrac{-3x^2}{2\sqrt{8 - x^3}}$ for $x < 2$.

(1 mark)

The graph of $y = f(x)$ is shown in Figure 9, along with the normal to the graph at $x = -1$.

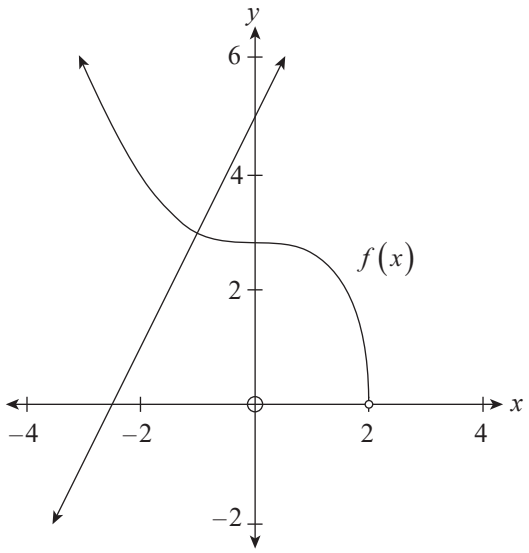

Figure 9

(b) (i) Show that the normal to the graph of $y = f(x)$ at $x = -1$ has the equation $2x - y = -5$.

(3 marks)

(ii) State the x-intercept of the normal $2x-y=-5$.

(1 mark)

(c) Consider the normal to the graph of $y=f(x)$ at $x=a$, where $a<2$.

(i) Show that the x-intercept of this normal is $x=a-\frac{3}{2}a^2$.

(4 marks)

(ii) Hence, using an algebraic approach, find the value of a such that the x-intercept of this normal is maximised.

(2 marks)