

Stage 1 Mathematical Methods

Trigonometry Test

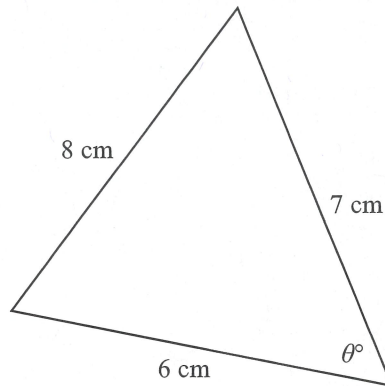
Topic 3: Subtopics 3.1, 3.2, 3.3

Total Marks – 44

(Calculator and one A4 page of handwritten notes permitted.)

Question 1 (4 marks)

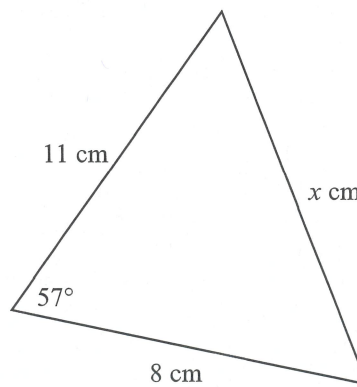
(a) Find the value of θ in the diagram (not drawn to scale).



$$\theta = \cos^{-1} \left(\frac{7^2 + 6^2 - 8^2}{2 \times 7 \times 6} \right)$$
$$= 75.5^\circ$$

(2 marks)

(b) Find the value of x in the diagram (not drawn to scale).

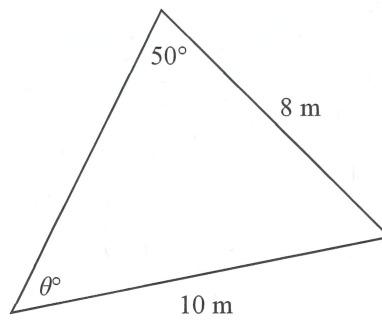


$$x = \sqrt{11^2 + 8^2 - 2 \times 11 \times 8 \cos 57^\circ}$$
$$= 9.44 \text{ cm}$$

(2 marks)

Question 2 (4 marks)

Find **ALL** possible values of θ in the diagram (not drawn to scale).



$$\frac{\sin \theta}{8} = \frac{\sin 50}{10}$$

$$\sin \theta = \frac{8 \sin 50}{10}$$

$$\theta = \sin^{-1}\left(\frac{8 \sin 50}{10}\right) \text{ or its supplement}$$

$$\theta_1 = 37.8^\circ$$

$$\theta_2 = 142.2^\circ$$

Check angle sum for θ_2 :

$$\theta_2 + 50^\circ = 192.2^\circ$$

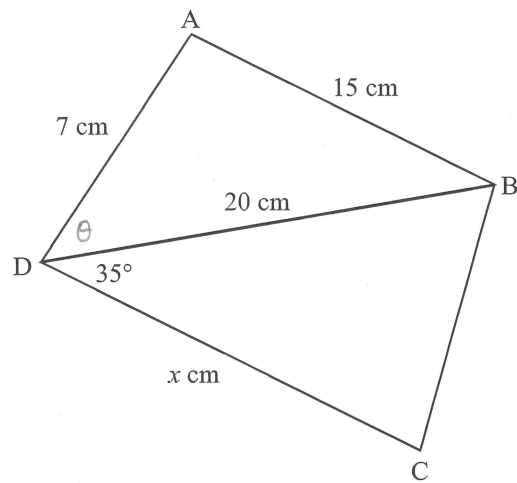
$$\geq 180^\circ$$

$\therefore \theta_2$ is not valid

(4 marks)

Question 3 (6 marks)

Consider the following diagram (not drawn to scale):



(a) Find the area of triangle ABD.

$$\theta = \cos^{-1} \left(\frac{20^2 + 7^2 - 15^2}{2 \times 20 \times 7} \right)$$
$$= 36.9^\circ$$
$$\text{Area } \triangle ABD = \frac{1}{2} \times 7 \times 20 \sin \theta$$
$$= 42 \text{ cm}^2$$

(4 marks)

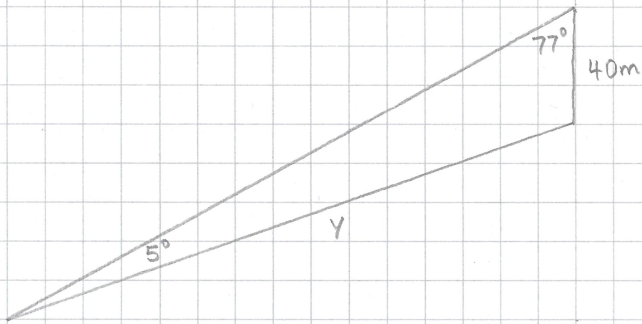
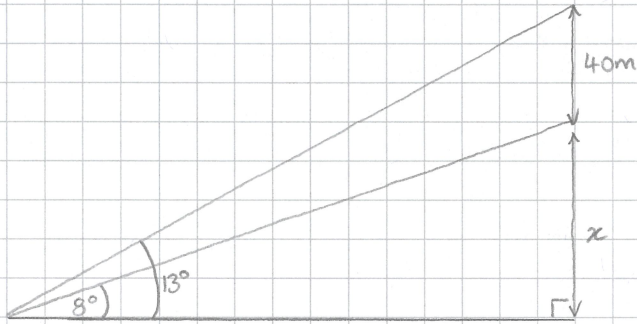
(b) Find the value of x given that the area of triangle BCD is 75 cm^2 .

$$75 = \frac{1}{2} \times 20x \sin 35^\circ$$
$$x = \frac{75}{0.5 \times 20 \sin 35^\circ}$$
$$= 13.08 \text{ cm}$$

(2 marks)

Question 4 (6 marks)

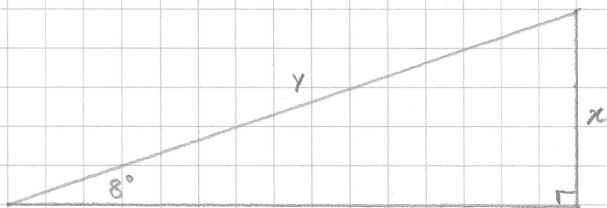
A tower 40 metres high, stands on top of a hill. From a point some distance from the base of the hill, the angle of elevation to the top of the tower is 13° . From the same point the angle of elevation to the bottom of the tower is 8° . Find the height of the hill.



$$\frac{y}{\sin 77^\circ} = \frac{40}{\sin 5^\circ}$$

$$y = \frac{40 \sin 77^\circ}{\sin 5^\circ}$$

$$y = 447.19\text{m}$$



$$\sin 8^\circ = \frac{x}{y}$$

$$x = y \cdot \sin 8^\circ$$

$$x = 62.24\text{m}$$

(6 marks)

Question 5 (2 marks)

(a) State the period of $y = 4 \sin\left(\frac{x}{5}\right)$.

$$T = 2\pi \div \frac{1}{5} = 10\pi$$

(1 mark)

(b) State the period of $y = \tan(3x) - 1$.

$$T = \frac{\pi}{3}$$

(1 mark)

Question 6 (2 marks)

(a) Find b given that the function $y = \sin(bx)$ has period $\frac{\pi}{4}$.

$$b = 2\pi \div \frac{\pi}{4} = 8$$

(1 mark)

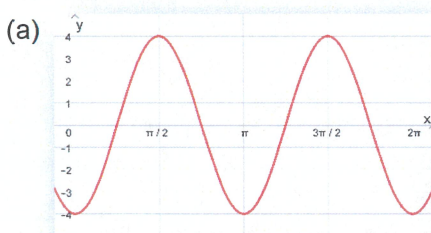
(b) Find b given that the function $y = \cos(bx)$ has period 4.

$$b = \frac{2\pi}{4} = \frac{\pi}{2}$$

(1 mark)

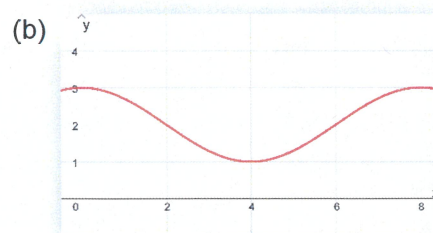
Question 7 (4 marks)

Find the cosine functions represented in each of the following graphs:



$$y = -4 \cos(2x)$$

(2 marks)



$$y = \cos\left(\frac{2\pi x}{8}\right) + 2$$

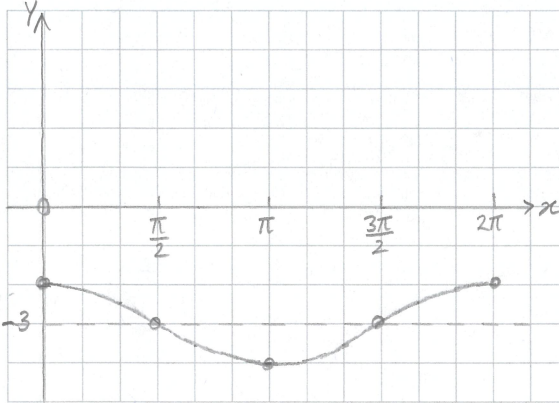
i.e. $y = \cos\left(\frac{\pi x}{4}\right) + 2$

(2 marks)

Question 8 (4 marks)

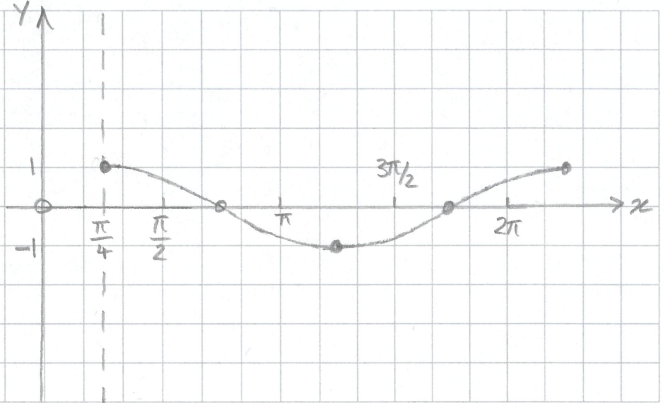
Sketch at least one complete cycle of the following graphs:

(a) $y = \cos(x) - 3$



(2 marks)

(b) $y = \cos(x - \frac{\pi}{4})$



(2 marks)

Question 9 (6 marks)

Solve the following algebraically for $0 \leq x \leq 2\pi$.

(a) $2 \sin(x) = -1$



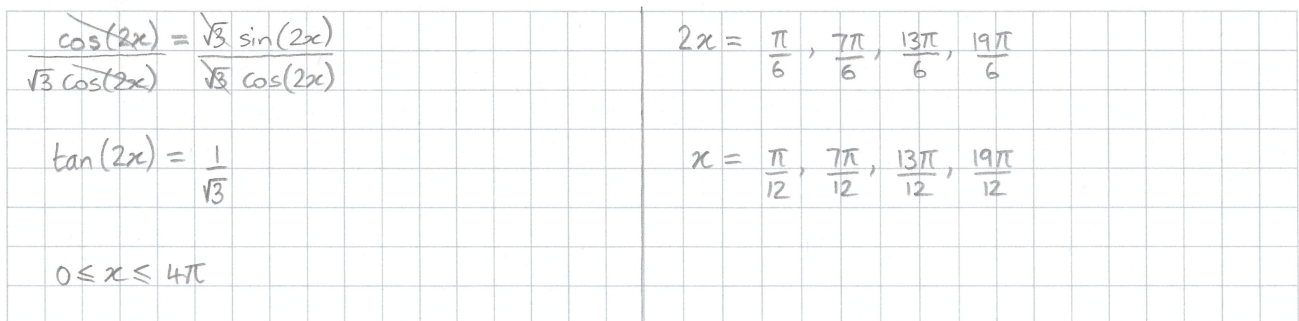
(2 marks)

(b) $2 \cos(x) - \sqrt{2} = 0$



(2 marks)

(c) $\cos(2x) = \sqrt{3} \sin(2x)$



(2 marks)

