Question 10 (16 marks)

(a) (i) State the roots of the complex equation $w^6 = 1$ in $r\,\text{cis}\,\theta$ form.

(2 marks)

(ii) On the Argand diagram in Figure 10, plot the roots identified in part (a)(i).

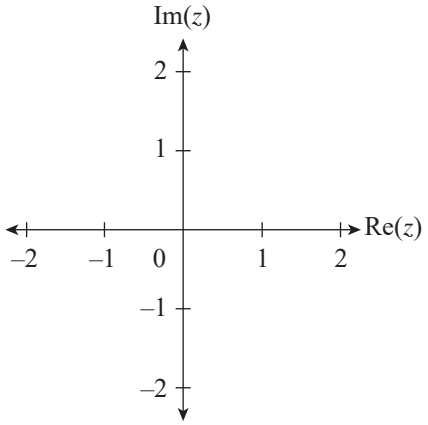

Figure 10

(2 marks)

(b) Consider the complex equation $(z-1)^6 = 1$.

(i) Using the roots identified in part (a)(i), state all the roots of the equation $(z-1)^6 = 1$, giving answers in any form.

(2 marks)

(ii) On the Argand diagram in Figure 11, plot the roots of the equation $(z-1)^6 = 1$.

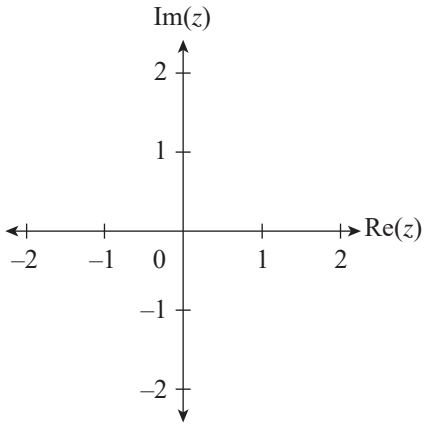

Figure 11

(1 mark)

(iii) Write the roots of $(z-1)^6 = 1$ in $r\,\mathrm{cis}\,\theta$ form or real form.

(2 marks)

(c) (i) Suppose that the polynomial $z^2 + bz + c$ has a zero $r\,\mathrm{cis}\,\theta$, where b and c are real, and $r > 0$ and $0 < \theta < \pi$.

Show that $b = -2r\cos\theta$ and $c = r^2$.

(2 marks)

(ii) Verify that $(z-1)^6 = z^6 - 6z^5 + 15z^4 - 20z^3 + 15z^2 - 6z + 1$.

(2 marks)

(d) Using part (b)(iii) and part (c), factorise $z^6 - 6z^5 + 15z^4 - 20z^3 + 15z^2 - 6z$ into the product of real linear and real quadratic factors.

(3 marks)