Question 3 (7 marks)

Let $P(x) = x^n + 5x^2 + cx - 1$, where n is a positive integer and c is a real constant.

(a) If $(x+1)$ is a factor of $P(x)$, show that c is equal to either 3 or 5.

$P(-1) = 0 \Rightarrow (-1)^n + 5 - c - 1 = 0$

$c = 4 + (-1)^n$

If n is odd, then $(-1)^n = -1$, and $c = 3$

If n is even, then $(-1)^n = 1$, and $c = 5$

(3 marks)

(b) When $P(x)$ is divided by $(x-2)$, the remainder is 57.
Show that there is only one possible value of n.

$P(2) = 57 \Rightarrow 2^n + 20 + 2c - 1 = 57$

$c = 5 \Rightarrow 2^n = 28$

$\therefore n = \log_2 28$ [which is not a positive integer]

$\therefore c \neq 5$

$c = 3 \Rightarrow 2^n = 32$

$\therefore n = 5$ [which is a positive integer]

(3 marks)

(c) Hence state the polynomial $P(x)$.

$P(x) = x^5 + 5x^2 + 3x - 1$

(1 mark)