

Question 3 (6 marks)

Let $P(x)$ be a real cubic polynomial.

- (a) If $2-i$ is a zero of $P(x)$, find a real quadratic factor of $P(x)$.

$$\begin{aligned} 2+i & \text{ must be another zero of } P(x) \\ \text{Sum of roots} & = 2+i + 2-i = 4 \\ \text{Product of roots} & = (2+i)(2-i) = 5 \\ \therefore x^2 - 4x + 5 & \text{ is a factor of } P(x) \end{aligned}$$

(2 marks)

- (b) When $P(x)$ is divided by $(x-1)$ the remainder is 6, and when $P(x)$ is divided by $(x-2)$ the remainder is 5.

Find $P(x)$.

$$\begin{aligned} P(x) & = (ax+b)(x^2-4x+5) \\ P(1) = 6 & \Rightarrow 2(a+b) = 6 \\ & \qquad \qquad a+b = 3 \quad \textcircled{1} \\ P(2) = 5 & \Rightarrow 2a+b = 5 \quad \textcircled{2} \\ \textcircled{2} - \textcircled{1} & \Rightarrow a = 2 \\ \text{Now } \textcircled{1} & \Rightarrow b = 1 \\ \therefore P(x) & = (2x+1)(x^2-4x+5) \end{aligned}$$

(4 marks)