Question 9 (7 marks)

$P(x)$ is a real cubic polynomial. When $P(x)$ is divided by $(x-1)$, the remainder is 35, and when it is divided by $(x+2)$, the remainder is 80.

(a) Find the values of a and b if $P(x)=Q(x)(x^2+x-2)+(ax+b)$.

$P(1)=35 \Rightarrow a+b=35$ ①

$P(-2)=80 \Rightarrow -2a+b=80$ ②

② − ① $\Rightarrow -3a=45$

$a=-15$

Now ① $\Rightarrow b=50$

(3 marks)

(b) (i) If $(x-2)$ is a factor of $P(x)$, show that $Q(2)=-5$.

$P(2)=0 \Rightarrow 4Q(2)-30+50=0$

$4Q(2)=-20$

$Q(2)=-5$

(1 mark)

(ii) If the leading coefficient of $P(x)$ is 1, show that $Q(x)=x-7$.

Let $Q(x)=x+c$

$Q(2)=-5 \Rightarrow 2+c=-5$

$c=-7$

$\therefore Q(x)=x-7$

(2 marks)

(iii) Hence find the expanded form of $P(x)$.

$P(x) = (x-7)(x^2+x-2) - 15x + 50$

$= x^3 + x^2 - 2x - 7x^2 - 7x + 14 - 15x + 50$

$= x^3 - 6x^2 - 24x + 64$

(1 mark)