Question 2 (6 marks)

(a) (i) Write each of the following complex numbers in polar form.

(1) $z = -\sqrt{2} + \sqrt{2}i$

$z = 2\,\text{cis}\left(\frac{3\pi}{4}\right)$

(1 mark)

(2) $w = \sqrt{6} - \sqrt{2}i$

$w = 2\sqrt{2}\,\text{cis}\left(-\frac{\pi}{6}\right)$

(1 mark)

(ii) Hence find zw in polar form.

$\therefore zw = 4\sqrt{2}\,\text{cis}\left(\frac{3\pi}{4} - \frac{\pi}{6}\right) = 4\sqrt{2}\,\text{cis}\left(\frac{7\pi}{12}\right)$

(1 mark)

(b) (i) Use de Moivre's theorem to write $(zw)^n$ in polar form, where n is a positive integer.

$(zw)^n = 2^{5n/2}\,\text{cis}\left(\frac{7n\pi}{12}\right)$

(1 mark)

(ii) Find the smallest positive value of n for which $(zw)^n$ is real and positive.

$\frac{7n\pi}{12} = k.2\pi$ where $k \in \mathbb{Z}$

$\therefore n = \frac{24k}{7}$

$k = 7 \Rightarrow n = 24$

(2 marks)