Question 2 (6 marks)

(a) (i) Express $1+i$ in exact polar form.

$\sqrt{2}\ \text{cis}\ \frac{\pi}{4}$

(1 mark)

(ii) Express $\sqrt{3}-i$ in exact polar form.

$2\ \text{cis}\left(-\frac{\pi}{6}\right)$

(1 mark)

(b) Use your answers to part (a) to show that $(1+i)\left(\sqrt{3}-i\right)=2\sqrt{2}\,\text{cis}\frac{\pi}{12}$.

$\therefore (1+i)(\sqrt{3}-i) = 2\sqrt{2}\ \text{cis}\left(\frac{\pi}{4}-\frac{\pi}{6}\right)$

$= 2\sqrt{2}\ \text{cis}\ \frac{\pi}{12}$

(2 marks)

(c) Hence show that the exact value of $\cos\frac{\pi}{12}$ is $\frac{\sqrt{6}+\sqrt{2}}{4}$.

$\therefore \sqrt{3} - i + i\sqrt{3} + 1 = 2\sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$

$\therefore 2\sqrt{2}\cos\frac{\pi}{12} = \sqrt{3}+1$

$\therefore 4\cos\frac{\pi}{12} = \sqrt{6}+\sqrt{2}$

$\therefore \cos\frac{\pi}{12} = \frac{\sqrt{6}+\sqrt{2}}{4}$

(2 marks)