**Question 3** (7 marks)

Let $z_1 = (1-i)^m$ and $z_2 = \left(1+i\sqrt{3}\right)^n$, where $m$ and $n$ are positive integers.

(a) Find $z_1$ and $z_2$ in $r\,\text{cis}\theta$ form.

$z_1 = \left[\sqrt{2}\,\text{cis}\left(-\frac{\pi}{4}\right)\right]^m = 2^{m/2}\,\text{cis}\left(-\frac{m\pi}{4}\right)$

$z_2 = \left[2\,\text{cis}\left(\frac{\pi}{3}\right)\right]^n = 2^n\,\text{cis}\left(\frac{n\pi}{3}\right)$

(3 marks)

(b) (i) If $z_1 = z_2$, show that $m = 2n$.

$|z_1| = |z_2| \Rightarrow \frac{m}{2} = n$

ie. $m = 2n$

(1 mark)

(ii) Hence find the smallest positive integers $m$ and $n$ such that $z_1 = z_2$.

$\arg z_1 = \arg z_2 \Rightarrow -\frac{m\pi}{4} + k.2\pi = \frac{n\pi}{3}$ where $k \in \mathbb{Z}$

$\therefore -\frac{2n\cancel{\pi}}{4} + k2\cancel{\pi} = \frac{n\cancel{\pi}}{3}$

$\therefore -6n + 24k = 4n$

$10n = 24k$

$n = \frac{12k}{5}$

If $k=5$ then $m=24$ and $n=12$

(3 marks)