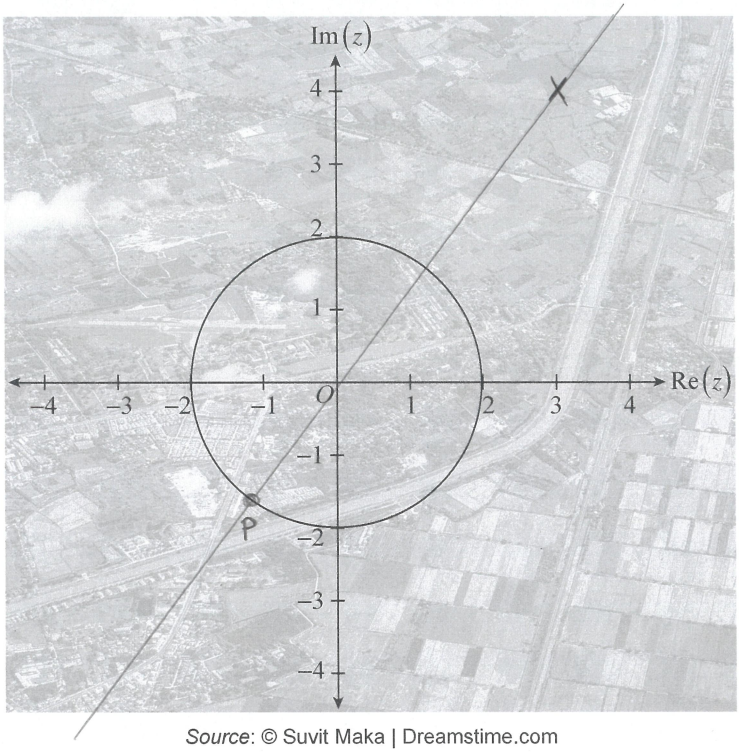
Question 10 (10 marks)

Figure 10 shows an Argand diagram superimposed on an aerial photograph.



Source: © Suvit Maka | Dreamstime.com

Figure 10

(a) Write an inequality that represents all complex numbers z in the region bounded by, and including, the circle.

$|z| \leq 2$

(2 marks)

(b) (i) On Figure 10, mark the position of the complex number $3 + 4i$ with an X. (1 mark)

(ii) Show that any point on the straight line through the origin (O) and X has the form $3t + 4it$, where t is a real parameter.

$\overrightarrow{OX} = 3 + 4i$

$t\overrightarrow{OX}$ describes any point on the straight line through O and X $(t \in \mathbb{R})$

ie. $3t + 4it$ where $t \in \mathbb{R}$

(1 mark)

(c) (i) If z is any point in the region bounded by, and including, the circle, apply the triangle inequality to the triangle that has vertices at z, X, and O to show that

$$|z-(3+4i)|\leq 7.$$

$XZ \leq OX + OZ$ by the triangle inequality

$\therefore |z-(3+4i)| \leq 5 + |z|$

$\leq 5 + 2$

$\therefore |z-(3+4i)| \leq 7$

(2 marks)

(ii) On Figure 10, on the region bounded by, and including, the circle, mark the point P for which $|z-(3+4i)|=7$. (1 mark)

(d) Using part (b)(ii) or otherwise, find the complex number that is represented by P.

$|3t + 4it| = 2$

$5|t| = 2$

$t = \pm \frac{2}{5}$

For point P, $t = -\frac{2}{5}$

$\therefore z_P = -\frac{2}{5}(3+4i)$

$= -\frac{6}{5} - \frac{8i}{5}$

(2 marks)

(e) A mobile phone tower at O provides reception for 2 km in any direction. A new tower is going to be built at X, which will provide reception for 7 km in any direction.

Explain why the tower at O will not be needed, once a tower is built at X.

P is the maximum distance away from X (7 units)

$\therefore$ all other possible values of z, on or inside the circle, are closer to X than 7 units

So, the tower at X will provide reception for all of the circular region radius 2 units centred at O

$\therefore$ The tower at O is not required

(1 mark)