**Stage 2 Specialist Mathematics**

**Complex Numbers and Real Polynomials Test**

**Topic 2: Subtopics 2.1, 2.2, 2.3, 2.4**

**Total Marks - 47**

**(Calculator and one A4 page of handwritten notes permitted.)**

1. (5 marks)

Let , where and are constants.

1. Given that is a factor of , show that .

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(2 marks)

1. When is divided by , the remainder is 6.

Show that .

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(1 mark)

1. Find and , and hence write as a product of linear factors.

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(2 marks)

1. (10 marks)
2. Let .

Show that has a factor of .

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(2 marks)

1. If is any polynomial of degree , prove that has a factor of .

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(2 marks)

1. is a real cubic polynomial with a zero of .
   1. Find a real quadratic factor of .

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(3 marks)

* 1. Find , given that has a factor of and that has a factor of .

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(3 marks)

1. (9 marks)

Let , where , , and are real numbers.

1. Given that is a zero of , show that is a quadratic factor of .

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(2 marks)

* 1. If is also a zero of , show that is a factor of degree 4 of .

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(2 marks)

1. Show that .

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(1 mark)

* 1. Given that is a factor of , explain why and .

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(1 mark)

1. If the remainder is 60 when is divided by , show that .

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(2 marks)

* 1. Using the information from parts (b) (ii) and (c) (i), solve for and , and hence find .

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(1 mark)

1. (8 marks)
2. On the Argand diagram below, show the set of complex numbers such that

(2 marks)

1. On the same Argand diagram, show the set of complex numbers such that

(2 marks)

1. Find in Cartesian form the exact value of all complex numbers that satisfy *both* the equation in part (a) *and* the equation in part (b).

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(4 marks)

1. (15 marks)
2. Write exactly in form, where and .

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(2 marks)

* 1. Hence find .

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(1 mark)

1. Solve , writing your answers exactly in form.

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(3 marks)

1. Show that .

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(1 mark)

1. Use your results from parts (b) and (c) to solve the equation

.

Write your answers exactly in form.

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(3 marks)

1. Plot your solutions from part (d) on the Argand diagram below, labelling them , , …, anticlockwise from the smallest positive argument.

(2 marks)

1. Find .

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(1 mark)

* 1. Explain why .

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(1 mark)

* 1. Find .

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