**Stage 2 Mathematical Methods**

**Discrete Random Variables Test**

**Topic 1: Subtopics 2.1, 2.2, 2.3**

**Total Marks – 40**

**(Calculator and one A4 page of handwritten notes permitted)**

1. (5 marks)
2. (i) Each time a fair dice is rolled, $X=1$ if the uppermost face is ‘3’ and $X=0$ otherwise.

On the basis of the information above, is $X$ a Bernoulli random variable? Tick the appropriate box.

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| --- | --- |
| Yes |  |
|  |  |
| No |  |

(1 mark)

 (ii) A fair coin is tossed five times. $X$ is the number of times that ‘heads’ is uppermost.

On the basis of the information above, is $X$ a Bernoulli random variable? Tick the appropriate box.

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| Yes |  |
|  |  |
| No |  |

(1 mark)

1. $X$ represents a Bernoulli random variable with probability of success $p=0.2$.

 (i) State the mean of $X$.

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(1 mark)

 (ii) Determine the standard deviation of $X$.

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(1 mark)

1. $X$ represents a Bernoulli random variable with probability of success $p$.

 State the value of $p$ for which the standard deviation of $X$ is maximised.

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(1 mark)

1. (8 marks)
2. The discrete random variable X has the following probability distribution:

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| --- | --- | --- | --- |
| $$x$$ | 0 | 9 | 11 |
| $$P\left(X=x\right)$$ | $$a$$ | $$\frac{1}{10}$$ | $$\frac{1}{15}$$ |

1. Find the value of $a$.

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(1 mark)

1. Find the mean, $μ\_{X}$.

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(2 marks)

1. Find the standard deviation, $σ\_{X}$.

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(2 marks)

1. People pay a fee to enter a local showground. Upon entry, each person receives 10 tokens that are redeemable for activities such as rides and games.

In one game, 30 balls numbered from 1 to 30 are placed into a bag. Players give two tokens to the game’s operator each time they randomly draw one numbered ball from the bag. The number on the ball is checked and the ball is put back into the bag. The number on the ball determines the outcome of the game, as shown in the table below:

|  |  |  |
| --- | --- | --- |
| *Number on the ball* | *Numbers of tokens won from the operator by the player* | *Probability of occurring* |
| a multiple of 9 | 9 | $$\frac{1}{10}$$ |
| a multiple of 11 | 11 | $$\frac{1}{15}$$ |

If the number on the ball drawn is neither a multiple of 9 nor a multiple of 11, the player wins no tokens from the operator.

1. Let $X$ represent the number of tokens won through playing this game.

What is the expected value of $X$?

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(1 mark)

1. In one day the game is played 1000 times.

Predict whether or not the operator will have more tokens at the end of the day than at the start of the day. Explain your answer.

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(2 marks)

1. (8 marks)

A seed wholesaler is selling a large quantity of seeds. From past experience, he knows that 80% of the seeds will germinate and grow into seedlings.

The manager of a plant nursery has purchased these seeds and is planning to germinate them in trays consisting of six pots (as shown in the photograph).

Only trays with one or more seedlings in all six pots can be sold.

The manager wants to make sure that more than 90% of trays can be sold.

1. The manager decides to plant two seeds in every pot.
2. Calculate the probability that, if two seeds are planted in a single pot, at least one of the seeds will germinate and grow into a seedling.

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(2 marks)

1. Using your answer to part (i), calculate the probability that, if two seeds are planted in each of the six pots in each tray, at least one of the seeds in each pot will germinate and grow into a seedling.

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(2 marks)

1. If three seeds are planted in each of the six pots in each tray, is it likely that more than 90% of trays could be sold? Show calculations to support your answer.

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(4 marks)

1. (10 marks)

Consider a multiple-choice question that contains five possible answers from which to choose. Only one of the five answers is correct.

1. A student attempts this multiple-choice question. Let $X=1$ if the student randomly chooses the correct answer and $X=0$ if the student randomly chooses an incorrect answer.
2. Construct a table of the probability distribution of $X$.

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(2 marks)

1. Calculate the mean of $X$.

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(1 mark)

1. A student attempts a test containing 50 of these multiple-choice questions. In order to pass the test, the student must correctly answer at least 25 of the questions.
2. The student knows the correct answer to 15 questions, and chooses these answers. The student randomly chooses an answer for each of the remaining 35 questions.
	1. What is the probability that the student passes the test?

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(2 marks)

* 1. In total, for this student, what is the expected number of correctly answered questions?

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(2 marks)

1. To have a probability of at least 33% of passing the test, what is the minimum number of questions to which the student must know the correct answer, given that the student will randomly guess the answers to the remaining questions?

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(3 marks)

1. (9 marks)

In Australia, mobile (cell) phone numbers contain 10 digits, beginning with the digits 04. An example of a mobile phone number is 0402 199 999.

Mobile phone numbers are allocated to telecommunications service providers, for distribution to their customers. This allocation is made according to the table below.

|  |  |
| --- | --- |
| *Telecommunications service provider* | *Allocated mobile phone numbers* |
| **Optus** (‘Optus Mobile Pty Limited’) | 0401 xxx xxx0402 xxx xxx0403 xxx xxx |
| **Vodafone** (‘Vodafone Australia Pty Limited’) | 0404 xxx xxx0405 xxx xxx0406 xxx xxx |
| **Telstra** (‘Telstra Corporation Limited’) | 0407 xxx xxx0408 xxx xxx0409 xxx xxx0400 xxx xxx |

A survey company has developed a machine that calls, at random, mobile phone numbers that begin with ‘040’.

Assume that all mobile phone numbers that begin with 040 are valid, and that customers have not reallocated their mobile phone numbers to another telecommunications service provider.

1. When the machine calls one mobile phone number:
2. what is the probability that this mobile phone number is allocated to Telstra?

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(1 mark)

1. what is the probability that this mobile phone number is ***not*** allocated to Telstra?

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(1 mark)

1. The machine calls batches of five mobile phone numbers.

Let $X$ be the number of these calls that are made to mobile phone numbers that are allocated to Telstra.

1. State the distribution of $X$.

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(1 mark)

1. Within one batch of five mobile phone numbers:
2. what is the probability that all five calls will be made to mobile phone numbers that are allocated to Telstra?

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(1 mark)

1. what is the probability that at least one call will be made to a mobile phone number that is allocated to Telstra?

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(2 marks)

1. How many mobile phone numbers would the machine need to call in order to have a minimum 99.5% chance of calling at least one mobile phone number that is allocated to Telstra?

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(3 marks)