**Question 15** (14 marks)

'Wacky Quackers' is a popular amusement park game in Australia. In this game, a player selects from a very large number of identical-looking plastic ducks that are floating on a pool of water. Each duck has the number $1$, $2$, $5$, or $10$ printed on its base. The numbers cannot be seen by the player when making their selection.

The number that is printed on the selected duck will be used to determine the player's prize. These numbers are distributed according to the table below.

| $x$ | 1 | 2 | 5 | 10 |
|---|---|---|---|---|
| $\Pr(X = x)$ | 0.5 | 0.2 | 0.2 | 0.1 |


*Source:* adapted from © Iprintezis | Dreamstime.com

(a) (i) Calculate $\mu_X$.

(2 marks)

(ii) Calculate $\sigma_X$.

(2 marks)

A player can pay $\$10$ and randomly select three ducks. This player wins a prize that has a dollar ($\$$) value equal to the sum of the numbers printed on the three ducks.


(b) (i) What is the probability that a player who randomly selects three ducks will win a prize that has a value of $\$30$?

(2 marks)

(ii) What is the probability that a player who randomly selects three ducks will win a prize that has a value of at least $\$5$?

(3 marks)

Let the discrete random variable $S_3$ represent the sum of the numbers printed on three randomly selected ducks.

(c) (i) Calculate $\mu_{S_3}$.

(1 mark)

(ii) Calculate $\sigma_{S_3}$.

(1 mark)

Alternatively, a player can pay $\$16$ and randomly select five ducks. This player wins a prize that has a dollar value equal to the sum of the numbers printed on the five ducks.


(d) Calculate the expected value of the sum of the numbers printed on the five randomly selected ducks.

(1 mark)

(e) Using your answers to part (c) and part (d), determine which *one* of the two options — three ducks for $\$10$ or five ducks for $\$16$ — will be more profitable for the owner of Wacky Quackers in the long run.

(2 marks)