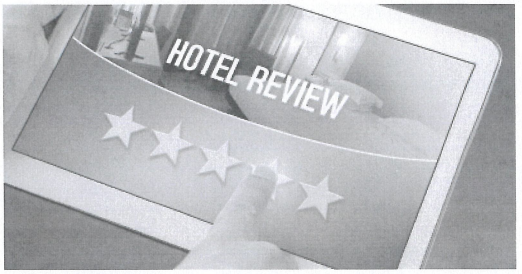
**Question 5** (9 marks)

A large number of guests of a hotel are required to anonymously rate their stay from 1 star to 5 stars. The best possible rating is 5 stars.

The distribution of the rating, $X$, given to the hotel by guests is shown in the table below.




*Source:* adapted from © Tero Vesalainen | dreamstime.com

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $\Pr(X = x)$ | 0.045 | 0.015 | 0.105 | 0.282 | $a$ |

(a) Show that $a = 0.553$.

$0.045 + 0.015 + 0.105 + 0.282 + a = 1$

$\therefore a = 0.553$

(1 mark)

(b) Determine the mean rating given to the hotel by its guests.

$\mu_X = 1 \times 0.045 + 2 \times 0.015 + 3 \times 0.105 + 4 \times 0.282 + 5 \times 0.553$

$= 4.28$ (3 s.f.)

(1 mark)

(c) Show that the probability that a randomly selected guest will rate the hotel less than 3 stars is 0.06.

$P(X < 3) = 0.045 + 0.015$

$= 0.06$

(1 mark)

Let $Y$ represent the number of guests from a random sample who rated the hotel less than 3 stars. $Y$ can be modelled using a binomial distribution.

(d) The hotel's managers randomly select a group of 20 guests. Calculate the probability that:

(i) exactly *two* guests rate the hotel less than 3 stars.

$Y \sim B(20, 0.06)$

$P(Y = 2) = 0.225$ (3s.f.)

(1 mark)

(ii) more than *five* guests rate the hotel less than 3 stars.

$P(Y > 5) = 8.69 \times 10^{-4}$

(2 marks)

(e) Managers are concerned that the hotel is receiving ratings of less than 3 stars and they decide to interview past guests.

How large a group of randomly selected guests is needed for the managers to have a greater than 90% chance of having at least *one* guest who has rated the hotel less than 3 stars?

$Y \sim B(n, 0.06)$

We require $P(Y \geq 1) > 0.90$

| $n$ | $P(Y \geq 1)$ |
|---|---|
| 37 | 0.8987 |
| 38 | 0.9048 |

Using technology, $n \geq 38$

(3 marks)