Question 13 (11 marks)

Frankie is an electrician based in Goodtown. She is considering introducing a new business model in which customers would pay a fixed price of 250 dollars ($) to purchase a ceiling fan and have it installed. Frankie knows that the actual cost of purchasing and installing a ceiling fan varies according to the type of material that the house is made from, as detailed in the table below.

Type of material	*Wood*	*Brick*	*Stone*	*Other*
Cost to electrician ($)	150	190	250	280

To investigate the likelihood of a long-term profit for this business model, Frankie carries out some research and finds that in Goodtown, 10% of houses are made from wood, 40% of houses are made from brick, and 20% of houses are made from stone.

Frankie assumes that Goodtown-based customers who contact her will live in 'randomly selected' types of house. Hence, let the amount of profit (in $) received by Frankie per house be represented by the discrete random variable X.

(a) Based on the information provided, complete the discrete probability distribution table below for X.

Type of material	*Wood*	*Brick*	*Stone*	*Other*
x	100	60	0	−30
$\Pr(X=x)$	0.1	0.4	0·2	0·3

(2 marks)

(b) (i) Hence calculate the value of $E(X)$.

$E(X) = 100\times0{\cdot}1 + 60\times0{\cdot}4 + 0\times0{\cdot}2 - 30\times0{\cdot}3$

$= 25$

(1 mark)

(ii) Interpret your answer to part (b)(i) in the context of the long-term profit.

In the long run, Frankie would expect to earn, on average, a profit of $25 per house.

(1 mark)

(c) Calculate σ_X.

$\sigma_x = \sqrt{100^2 \times 0.1 + 60^2 \times 0.4 + 0^2 \times 0.2 + (-30)^2 \times 0.3 - 25^2}$

$= 45.7$ (3s.f.)

(1 mark)

(d) Frankie can install a maximum of 15 ceiling fans per week during the 48 working weeks in 1 year.

(i) What is Frankie's maximum expected yearly profit from installing ceiling fans?

$15 \times 48 \times \$25 = \$18,000$

(1 mark)

(ii) Give *one* reason why Frankie's maximum expected yearly profit might statistically differ from her actual yearly profit.

The demand for ceiling fans may depend upon the type of material a house is constructed from

(1 mark)

(e) Frankie requires a profit of at least \$50 000 per year in order for this business model to be worthwhile.

(i) What fixed price would Frankie need to charge, in order for the business model to be worthwhile?

Additional profit required $= 50,000 - 18,000 = \$32000$

Additional profit per house $= \frac{32,000}{15 \times 48} = \$44.40$ (3s.f.)

$\therefore$ Fixed price $= 250 + 44.40 = \$295$ (3s.f.)

(3 marks)

(ii) Suggest *one* change that Frankie could make to the business model in order to increase her expected yearly profit (other than changing the fixed price).

Instead of a fixed price for all types of construction materials, charge a different price for each type of construction material.

(1 mark)