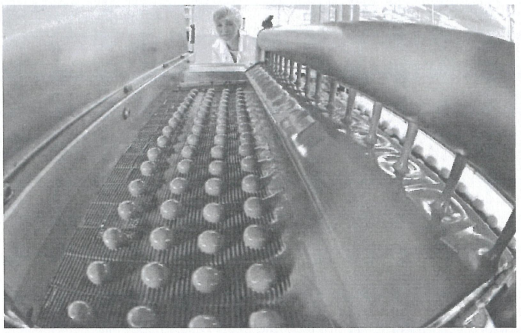
Question 8 (16 marks)

A company produces a type of fruit-filled chocolate called Choc-pricots. They are produced on an automated assembly line that can result in imperfect Choc-pricots.



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(a) The probability is 0.15 that a Choc-pricot randomly selected off the assembly line is imperfect.

Use the binomial distribution to determine the probability that, in a random sample of 80 Choc-pricots:

(i) *exactly* 6 Choc-pricots will be imperfect.

$X \sim B(80, 0.15)$

$P(X = 6) = 0.0205$ (3s.f.)

(1 mark)

(ii) *more than* 20 Choc-pricots will be imperfect.

$P(X > 20) = 0.00631$ (3s.f.)

(2 marks)

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The manager of the assembly line can adjust the rate of production of Choc-pricots, resulting in a change in the probability that a randomly selected Choc-pricot is imperfect. This probability can be modelled by the function

$$p(x) = \sin\left(0.00392x^2\right) + e^{-x}, \text{ for } 0 < x \leq 20,$$

where $p(x)$ represents the probability that a randomly selected Choc-pricot is imperfect, and x represents the rate of production measured in Choc-pricots per second.

(b) *On the axes in Figure 8*, sketch the graph of $y = p(x)$.

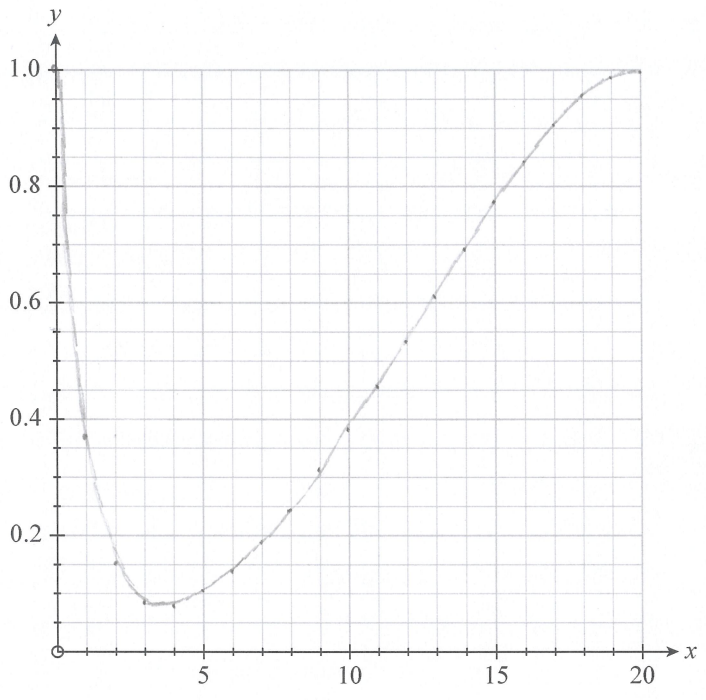

Figure 8

(3 marks)

(c) Using the function $p(x)$, determine the possible rate(s) of production that resulted in the given probability of 0.15 stated in part (a).

$x = 2.01$ or $x = 6.15$ (3s.f.)

(2 marks)

(d) The minimum probability that a randomly selected Choc-pricot is imperfect, according to the function $p(x)$, is 0.0781 (correct to three significant figures).

State the corresponding rate of production that would result in this probability.

$x = 3.58$ (3s.f.)

(1 mark)