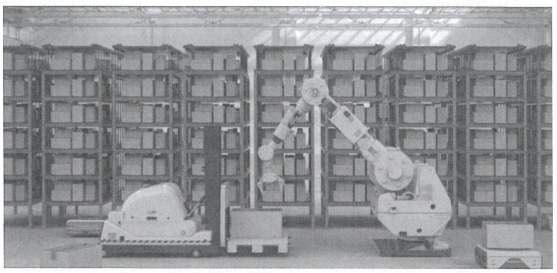
Question 3 (7 marks)

A particular warehouse uses robotic workers initially to retrieve all products that are requested by customers. However, the probability that a robotic worker retrieves an incorrect product is 10%. A human worker is then required to retrieve the correct product.



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(a) Let X be the number of times, from 600 randomly selected customer requests, that a human worker is required to retrieve the correct product.

(i) State *one* condition that needs to be true in order for X to be modelled by a binomial distribution.

There are a fixed number of independent trials, each with the same probability of success.

(1 mark)

Assume that X can be modelled by a binomial distribution.

(ii) Calculate the expected number of times, from 600 randomly selected customer requests, that a human worker will be required to retrieve the correct product.

$X \sim B(600, 0.10)$

$E(X) = 600 \times 0.10$

$= 60$

(1 mark)

(iii) Determine the probability that, from 600 randomly selected customer requests, a human worker will be required more than 80 times to retrieve the correct product.

$P(X > 80) = 0.00365$ (3s.f.)

(2 marks)

(b) After a robotic worker has retrieved an incorrect product, a human worker always retrieves the correct product on the first attempt.

The time taken to retrieve a product is, on average:

- 7 minutes for a robotic worker
- 12 minutes for a human worker.

Let T be the random variable that represents the *total* time taken (in minutes) to retrieve the correct product. A simple model for T is shown in the discrete probability distribution table below.

	Robotic worker retrieves correct product	*Human worker retrieves correct product*
Total time taken (t)	7	19
Probability $\Pr(T=t)$	0.90	0.10

(i) Explain how the value of 19 in the table above was obtained, in the context of the problem.

The total time to retrieve the correct product equals the time taken by the robot to retrieve the incorrect product plus the time taken by a human to retrieve the correct product.

So $t = 7 + 12 = 19$ minutes

(1 mark)

(ii) Calculate the expected value of the *total* time taken to retrieve the correct product, using the information in the table above.

$E(T) = 7 \times 0.90 + 19 \times 0.10$

$= 8.2$ minutes

(1 mark)

(c) One statistician claims that an improved model for T can be developed using a probability density function.

Justify the statistician's claim.

Time is a continuous random variable, so a probability density function is appropriate

(1 mark)