

Question 12 (16 marks)

The distributor of a particular seed claims that, on average, 60% of the seeds are viable, meaning that they will grow into seedlings.

- (a) Let X be the random variable representing the number of viable seeds per planting of eight seeds.

- (i) State the distribution of X .

(1 mark)

- (ii) Out of repeated plantings of eight seeds at a time, what is the expected number of viable seeds?

$$E(X) = 8 \times 0.60$$
$$= 4.8$$

(1 mark)

- (iii) The table below shows the probability of $k = 0, 1, 2, 3 \dots$ seeds being viable. Complete the table for $k = 4, 5, 6$.

k	$P(X = k)$
0	0.0007
1	0.0079
2	0.0413
3	0.1239
4	0.2322
5	0.2787
6	0.2090
7	0.0896
8	0.0168

(2 marks)

(b) A gardener plants eight of these seeds in his garden.

(i) What is the most likely number of these seeds that will be viable?

(1 mark)

(ii) What is the probability that six or fewer of these seeds will be viable?

$$P(X \leq 6) = 0.894 \text{ (3 s.f.)}$$

(1 mark)

(c) A farmer bought a large bag of these seeds. She planted 100 of the seeds from this bag and observed that 53 of these seeds were viable.

(i) Calculate the proportion of planted seeds that were viable.

(1 mark)

(ii) Calculate a 95% confidence interval for the proportion of seeds in the bag that are viable.

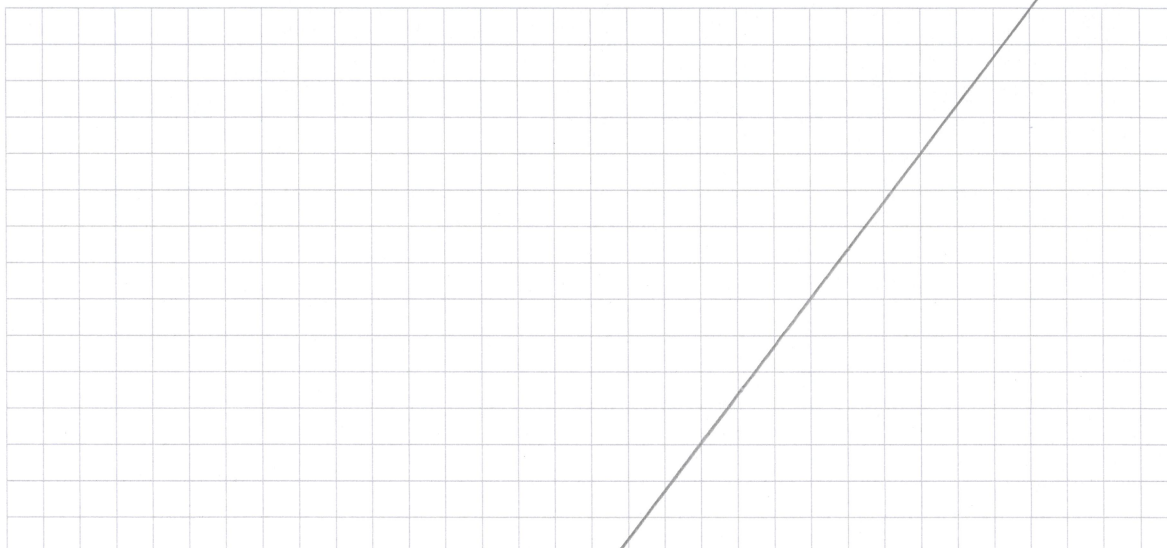
(2 marks)

(iii) Based on the confidence interval that you calculated in part (c)(ii), can it be concluded that the distributor's claim that 'on average, 60% of the seeds are viable' is false? Explain your answer.

(2 marks)

- (iv) Given an estimate p^* , show that the sample size n required to obtain a 95% confidence interval of width w for the population proportion is

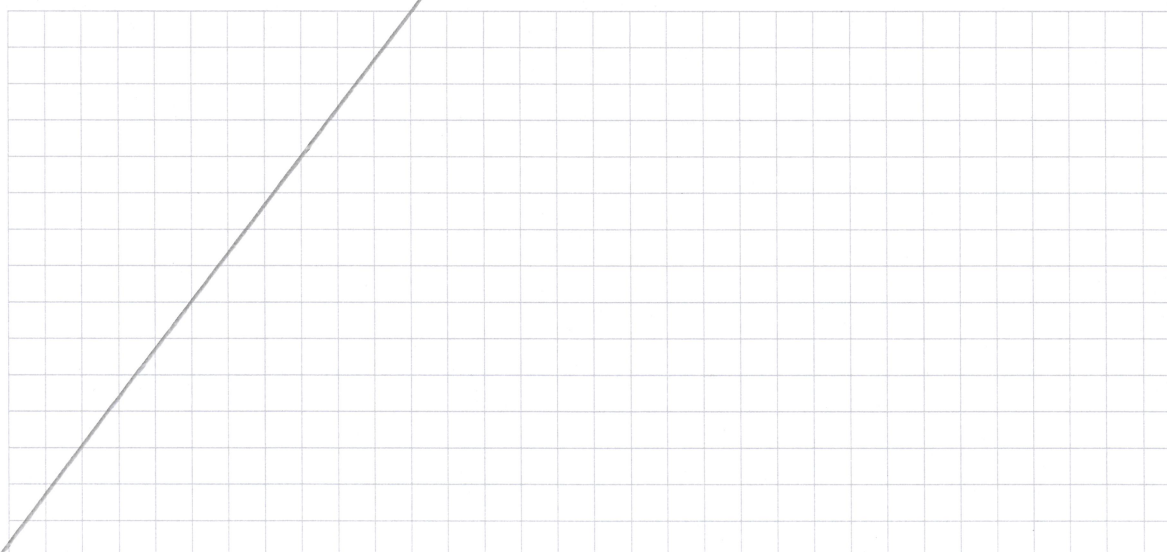
$$n = \left(\frac{2 \times 1.96}{w} \right)^2 p^* (1 - p^*).$$



(3 marks)

- (v) The farmer wishes to reduce the uncertainty in the estimate of the proportion of seeds in her bag that are viable.

Using the information provided in part (c)(iv), or otherwise, calculate the smallest number of seeds that she would need to plant in order to have a confidence interval with width 0.1.



(2 marks)