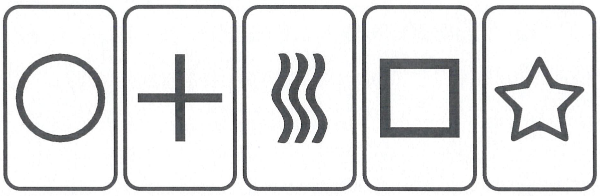
Question 14 (12 marks)

Extrasensory perception (ESP) is defined as the reception of information without the use of the recognised physical senses such as sight or hearing.

One test for ESP uses a set of Zener cards. A set of 25 Zener cards contains five of each of the cards illustrated below. In this test, a test administrator randomly selects one card from the set of 25 cards. The card is concealed from the test subject, who attempts to correctly identify the symbol that is printed on the card. The test administrator records whether or not the test subject correctly identifies the symbol printed on the card.



Source: adapted from Ryazanov, M 2014, 'File: Zener cards', *Wikimedia Commons, the free media repository*, viewed 28 August 2018, https://commons.wikimedia.org/wiki/File:Zener_cards_(color).svg

(a) Explain why the test subject's attempt to correctly identify which of the five symbols is printed on a selected card constitutes a Bernoulli trial.

There are only two possible outcomes: "success" if they correctly identify the card and "failure" if they do not.

(1 mark)

To complete the test, the process described above is undertaken 25 times, with cards being returned to the set between each attempt.

Let X represent the number of correct identifications out of 25 attempts, where the test subject is identifying symbols at random with the probability of correct identification $p = 0.2$.

(b) What is the probability that the test subject makes:

(i) exactly five correct identifications?

$X \sim B(25, 0.2)$

$P(X = 5) = 0.196$ (3s.f.)

(1 mark)

(ii) no more than seven correct identifications?

$P(X \leq 7) = 0.891$ (3s.f.)

(2 marks)

(c) In the context of this test, write a probability statement that is equivalent to:

(i) $C_8^{25}(0.2)^8(0.8)^{17} = 0.0623$.

$P(X = 8) = 0.0623$

(1 mark)

(ii) $1 - C_0^{25}(0.2)^0(0.8)^{25} = 0.996$.

$P(X \geq 1) = 0.996$

(1 mark)

To be successful in this test, the test subject must make at least k correct identifications, where k is chosen so that the chance of being successful by the random identification of symbols is less than 2.5%.

(d) Calculate k^*, the minimum value of k. Support your answer with probability calculations correct to four significant figures.

We require $P(X \geq k) < 0.025$

k	$P(X \geq k)$
8	0.1091
9	0.0468
10	0.0173

Using technology, $k^* = 10$

(2 marks)

 PLEASE TURN OVER

Students at a high school watched a television program investigating ESP. One student, Ari, undertook an ESP test using Zener cards. She made k^* correct identifications — where k^* is the value calculated in part (d) — and claimed to possess ESP.

(e) Based on the information in part (d), what can be said about Ari's claim?

Ari's claim is not necessarily true.
You would expect a small proportion of students (about 1.73%) to make at least ten correct identifications by just randomly guessing.

(1 mark)

(f) If 624 students at this high school undertook an ESP test using Zener cards and were identifying symbols at random, what is the probability that one or more of these students would make at least k^* correct identifications?

Let Y = the number of students who make at least ten correct identifications

$Y \sim B(624, 0.0173)$

$P(Y \geq 1) = 1.00$ (3s.f.)

(2 marks)

The principal of the school suggests that Ari was identifying symbols at random and hence does not possess ESP.

(g) Explain how your answer to part (f) supports the principal's suggestion.

It is almost 100% certain that at least one student (out of 624 students) would make at least ten correct identifications by just randomly guessing.

(1 mark)