Question 14 (16 marks)

(a) (i) Show that $\sin x + \cos x = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$.

(1 mark)

(ii) Figure 16 shows the graph of $g(x) = \sin x + \cos x$ for $-\pi \le x \le \pi$.

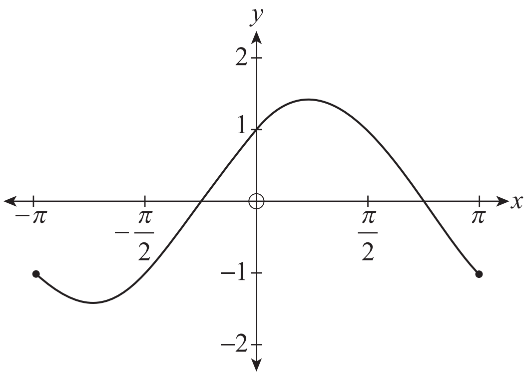

Figure 16

Explain why $g(x)$ is a function, but does not have an inverse function.

(2 marks)

(iii) Explain why the following function *does* have an inverse function:

$$f(x) = \sin x + \cos x \text{ where } -\frac{3\pi}{4} \le x \le \frac{\pi}{4}.$$

(1 mark)

(iv) Show that $f^{-1}(x) = \arcsin\left(\dfrac{x}{\sqrt{2}}\right) - \dfrac{\pi}{4}$.

(2 marks)

(b) Figure 17 shows the graph of the inverse function $f^{-1}(x)$.

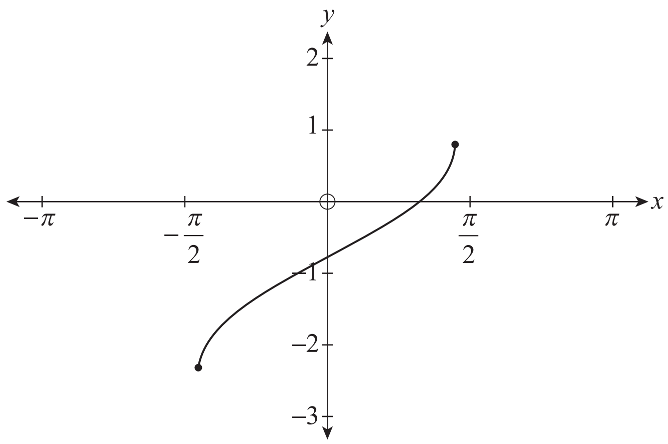

Figure 17

State the domain and range of $f^{-1}(x)$ in exact form.

(2 marks)

(c) If $y = \arcsin\left(\frac{x}{\sqrt{2}}\right)$, then $\sin y = \frac{x}{\sqrt{2}}$.

Using implicit differentiation, show that

$$\frac{dy}{dx} = \frac{\frac{1}{\sqrt{2}}}{\sqrt{1 - \frac{x^2}{2}}}.$$

(3 marks)

Consider a wall brace leaning against a building. The bottom of the wall brace is 5 metres along the ground from the base of the building, and the top of the wall brace is 5 metres above the ground, as shown in Figure 18.

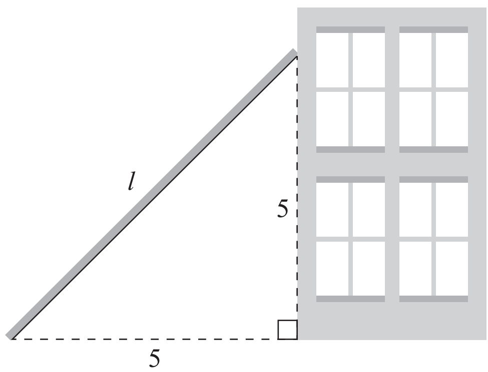

Figure 18

(d) Show that the length of the wall brace, l, is $5\sqrt{2}$ metres.

(1 mark)

The top of this wall brace slides down the side of the building. The top of the wall brace is now x metres above the ground, and θ is the angle of inclination of the wall brace with respect to the ground, as shown in Figure 19.

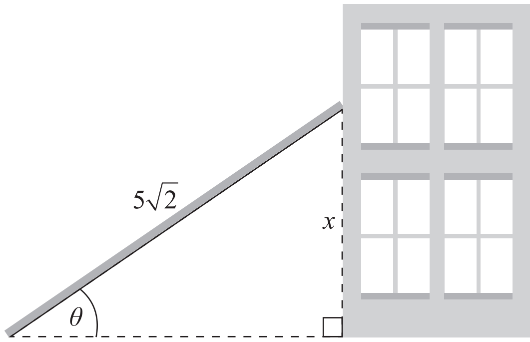

Figure 19

(e) (i) Show that $\frac{d\theta}{dt} = \frac{\frac{1}{5\sqrt{2}}\frac{dx}{dt}}{\sqrt{1-\frac{x^2}{50}}}$.

(2 marks)

(ii) If the top of the wall brace slides down the side of the building at a rate of 0.05 metres per second, at what rate is θ changing when the bottom of the wall brace is 6 metres along the ground from the base of the building?

(2 marks)