QUESTION 15   (15 marks)

(a) Consider the equation $x^3 + x^2 + x + 1 = 0$.

    (i)   Show that $x = -1$ is a root of this equation.

(1 mark)

    (ii) Show that there are no other real roots.

(1 mark)

(b) Let $g(x) = \sqrt{x}$ and $h(x) = x^3 + x^2 + x + 1$.

    (i)   Find the composite function $g(h(x))$.

(1 mark)

    (ii) State the domain of $g(h(x))$.

(1 mark)

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(c) Consider $f(x) = \sqrt{x^3 + x^2 + x + 1}$.

(i) Draw the graph of $y = f(x)$ on the axes in Figure 17.

Figure 17 (2 marks)

(ii) Explain why $f(x)$ has an inverse, $f^{-1}(x)$.

(1 mark)

(iii) Using your answer to part (c)(i), sketch the graph of $f^{-1}(x)$ on the axes in Figure 18.

Figure 18 (2 marks)

(iv) (1) Find $f(1)$.

(1 mark)

(2) Find $f^{-1}(2)$.

(1 mark)

(d) If $y = f^{-1}(x)$, then $x = f(y)$.

(i) Use implicit differentiation to show that $\dfrac{d}{dx}\left(f^{-1}(x)\right) = \dfrac{dy}{dx} = \dfrac{1}{f'(y)}$.

(1 mark)

(ii) Find $\dfrac{d}{dx}\left(f^{-1}(x)\right)$ at $x = 2$.

(3 marks)

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