

**Question 4** (10 marks)

Consider the function  $f(x) = \frac{x^3 - 2x + 5}{x^2 + 1}$ .

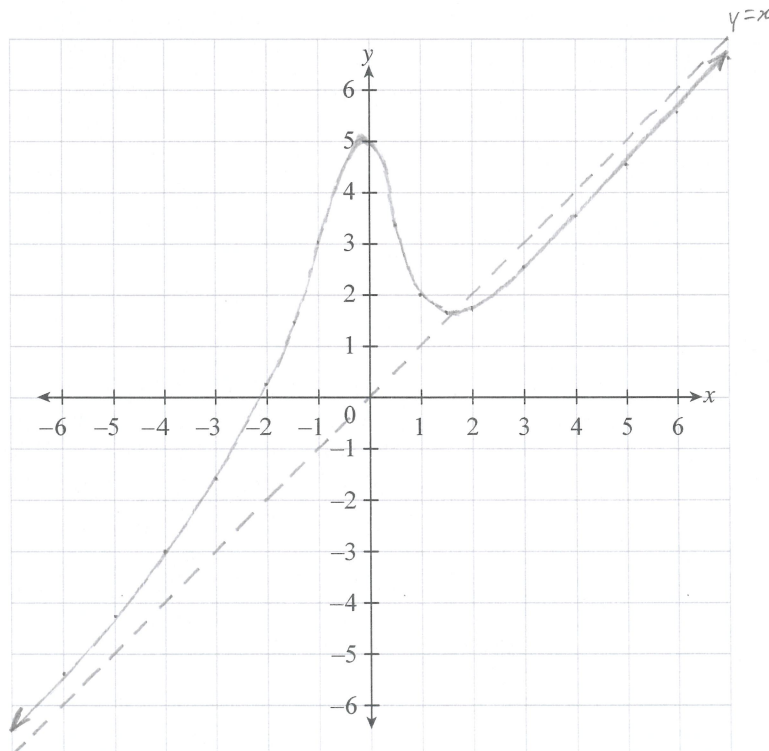
(a) Use a division process to show that  $f(x) = x - \frac{3x - 5}{x^2 + 1}$ .

$\begin{array}{r} x^2 + 1 \overline{) x^3 + 0x^2 - 2x + 5} \\ \underline{x^3 \phantom{+ 0x^2} + x} \phantom{+ 5} \\ -3x + 5 \end{array}$
$\therefore f(x) = x + \frac{5 - 3x}{x^2 + 1} = x - \frac{3x - 5}{x^2 + 1}$

(2 marks)

(b) On the axes in Figure 3, draw the function  $f(x) = x - \frac{3x - 5}{x^2 + 1}$ .

Clearly show the behaviour of the function near any asymptotes.



**Figure 3**

(3 marks)

(c) Find  $g(f(x))$ , given  $g(x) = \sqrt{x}$  and  $f(x) = x - \frac{3x-5}{x^2+1}$ .

$$g(f(x)) = \sqrt{x - \frac{3x-5}{x^2+1}}$$

(1 mark)

The graph of  $y = g(f(x))$  is shown in Figure 4.

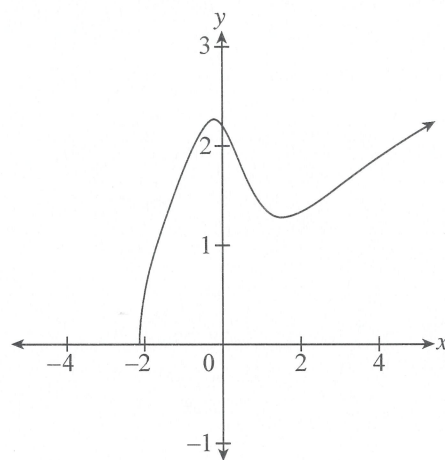
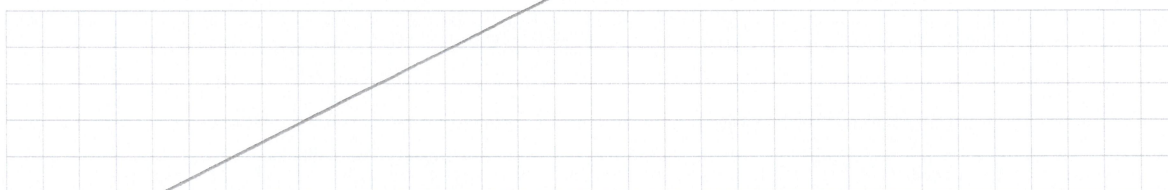


Figure 4

(d) Consider the solid obtained by rotating the graph of  $y = g(f(x))$  about the  $x$ -axis between  $x = -1$  and  $x = 1$ .

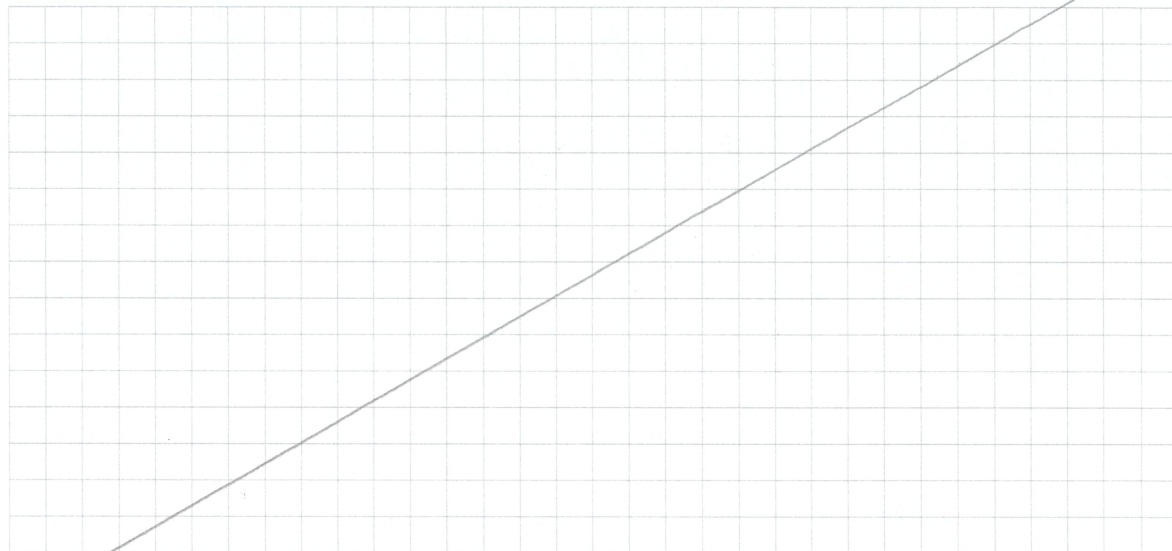
(i) Show that the volume of this solid is given by the equation

$$V = \pi \int_{-1}^1 \left( x - \frac{3x}{x^2+1} + \frac{5}{x^2+1} \right) dx.$$



(1 mark)

(ii) Show that the **exact** volume of this solid is  $\frac{5\pi^2}{2}$ .



(3 marks)