Question 13 (15 marks)

Figure 12 shows the graph of $g(x) = \arcsin(x^2)$ for $-1 \le x \le 1$.

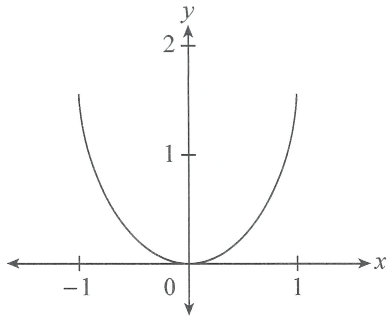

Figure 12

(a) (i) Explain why $g(x)$ does not have an inverse.

$g(x)$ does not satisfy the horizontal line test (it is a many-to-one function)

$\therefore$ $g(x)$ does not have an inverse

(1 mark)

(ii) Explain why $f(x) = \arcsin(x^2)$ for $0 \le x \le 1$ does have an inverse.

$f(x)$ satisfies the horizontal line test (it is a one-to-one function)

$\therefore$ $f(x)$ has an inverse

(1 mark)

(b) Figure 13 shows the graph of $y = f(x)$.

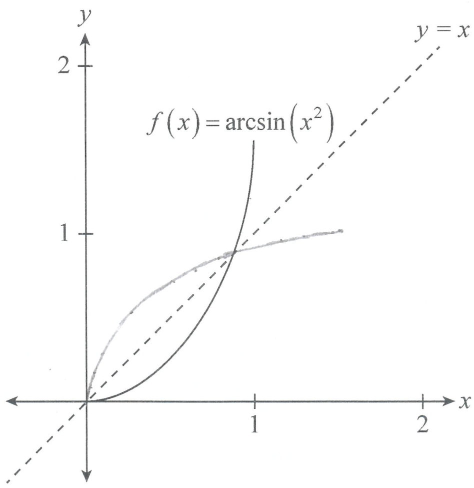

Figure 13

(i) On the axes in Figure 13, sketch the graph of $f^{-1}(x)$. (2 marks)

(ii) Write the exact domain of $f^{-1}(x)$.

$0 \leq x \leq \frac{\pi}{2}$

(1 mark)

(iii) Find the equation of $f^{-1}(x)$.

$f(x)$: $y = \arcsin(x^2)$

$f^{-1}(x)$: $x = \arcsin y^2$

$\sin x = y^2$

$y = \sqrt{\sin x}$

$f^{-1}(x) = \sqrt{\sin x}$

(2 marks)

(c) If $y = \arcsin(x^2)$, then $x^2 = \sin y$.

Hence use implicit differentiation to show that $\frac{dy}{dx} = \frac{2x}{\sqrt{1-x^4}}$.

(3 marks)

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(d) (i) Use integration by parts to show that

$$\int x \arcsin\left(x^2\right) \mathrm{d}x = \frac{1}{2}x^2 \arcsin\left(x^2\right) + \frac{1}{2}\sqrt{1-x^4} + c$$

where c is a constant.

(3 marks)

(ii) Hence find the exact value of $\int_0^1 x \arcsin\left(x^2\right) \mathrm{d}x$.

(2 marks)