**KEY FACTS AND CONCEPTS**

**Functions and sketching graphs**

* The **domain** of a function is the set of all possible *input* values for that function.
* The **range** of a function is the set of all possible *output* values for that function.
* If and are two functions, then the **composite function** if this exists.
* **One-to-one functions** satisfy both the vertical line test and the horizontal line test.
* **Many-to-one functions** satisfy the vertical line test but not the horizontal line test.
* If a function is one-to-one, it *will* have an **inverse function** which we denote .
* If a function is many-to-one, it *will not* have an inversefunction.
* If has an inverse function, then :
  + Has a graph which is the reflection of in the line .
  + Satisfies and .
* The **domain of** is equal to the range of .
* The **range of**  is equal to the domain of .
* Any function which has an inverse, and whose graph is symmetrical about the line is a **self-inverse-function** with .
* A **reciprocal function** is a function of the form , where is a constant.
* The **reciprocal of a function** is .
* When is graphed from :
  + The zeros of become vertical asymptotes of .
  + The vertical asymptotes of become zeros of .
  + The local maxima of become local minima of .
  + The local minima of become local maxima of .
  + .
  + .
  + .
  + .
* A **rational function** is a function of the form where and are polynomials.
* The **vertical asymptotes** of correspond to the zeros of .
* The **non-vertical asymptotes** of can be found by first using polynomial division and then considering the behaviour of as .
* The **absolute value** or **modulus** of is
* The following are **properties of the absolute value**:
  + for all .
  + for all .
  + for all and , .
  + for all .
  + for all and .
  + for all and .
  + .
  + .
* To obtain the **graph of**  from the graph of :
  + Discard the graph for .
  + Reflect the graph for in the -axis, keeping what was there.
  + Points on the -axis are invariant.
* To obtain the **graph of**  from the graph of :
  + Keep the graph for .
  + Reflect the graph in the -axis for , discarding what was there.
  + Points on the -axis are invariant.
* To **solve** **equations** **and** **inequalities** involving absolute value functions it is useful to remember that:
  + If where , then .
  + If then .