|  |
| --- |
| **VECTORS** |

**KEY FACTS AND CONCEPTS**

**5A VECTORS IN SPACE**

* The **position vector** of from where is   
  .
* The **magnitude** or **length** of the vector is  
  .
* Two vectors are **equal** if they have the same magnitude and direction. If and , then  
  .

**5B OPERATIONS WITH VECTORS IN SPACE**

* **Operations** with vectors:
* for any scalar

**5C VECTOR ALGEBRA**

* **Properties** of vectors:
  + {commutative property}
  + {associative property}
  + {additive identity}
  + {additive inverse}
  + {distributive property}
  + where is parallel to
  + {triangle inequality}

**5D THE VECTOR BETWEEN TWO POINTS**

* If and are two points in space then the **position vector of relative to** is  
  .
* Distance .

**5E PARALLELISM**

* If is **parallel** to , then there exists a scalar such that   
  .
* If for some scalar , then is parallel to , and .
* The unit vector (length 1 unit) in the direction of is .
* , , and are **collinear** if for some scalar .

**5F THE SCALAR PRODUCT OF TWO VECTORS**

* If and , the **scalar product** of and is defined as .
* **Properties**:

**5G THE ANGLE BETWEEN TWO VECTORS**

* The angle between two vectors and can be found using  
  .
* For non-zero vectors and :  
   and are **perpendicular**.
* and are non-zero **parallel vectors**.

**5H PROOF USING VECTOR GEOMETRY**

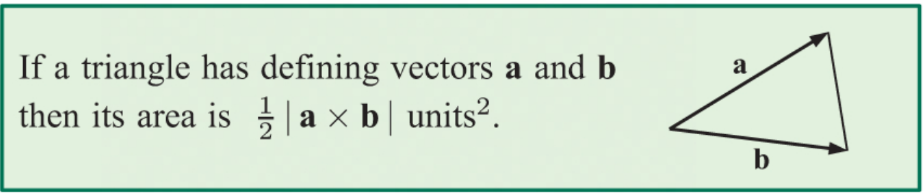
**5I THE VECTOR PRODUCT OF TWO VECTORS**

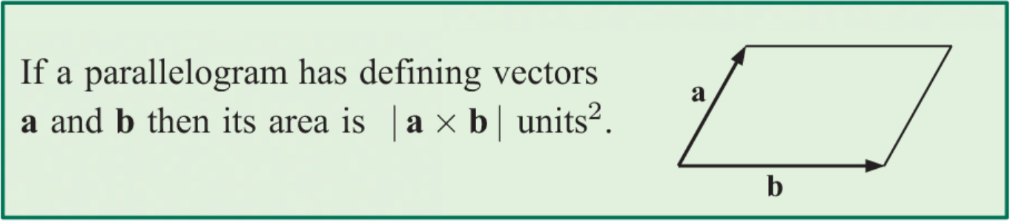
* If and , the **vector product** of and is defined as .
* **Properties**:
  + is a vector which is perpendicular to both and
  + for all
  + for all and
  + The direction of is determined by the **right hand rule**
  + The length of is where is the angle between and
  + If is a unit vector in the direction of then
  + For non-zero and , is parallel to
  + is called the **scalar triple product**

|  |
| --- |
| **VECTOR APPLICATIONS** |

**KEY FACTS AND CONCEPTS**

**6A AREA**





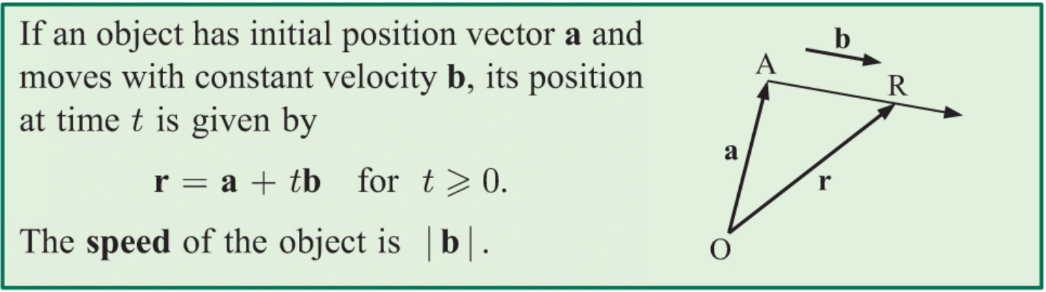
**6B LINES IN 2 AND 3 DIMENSIONS**

* If a **straight line** passes through the point with position vector  
   and has direction vector then:
* Its **vector equation** is
* Its **parametric equations** are
* Its **Cartesian equations** are

**6C THE ANGLE BETWEEN TWO LINES**

* The **acute angle between two lines** with direction vectors and is
* The two lines are **parallel** if for some scalar
* The two lines are perpendicular if

**6D CONSTANT VELOCITY PROBLEMS**



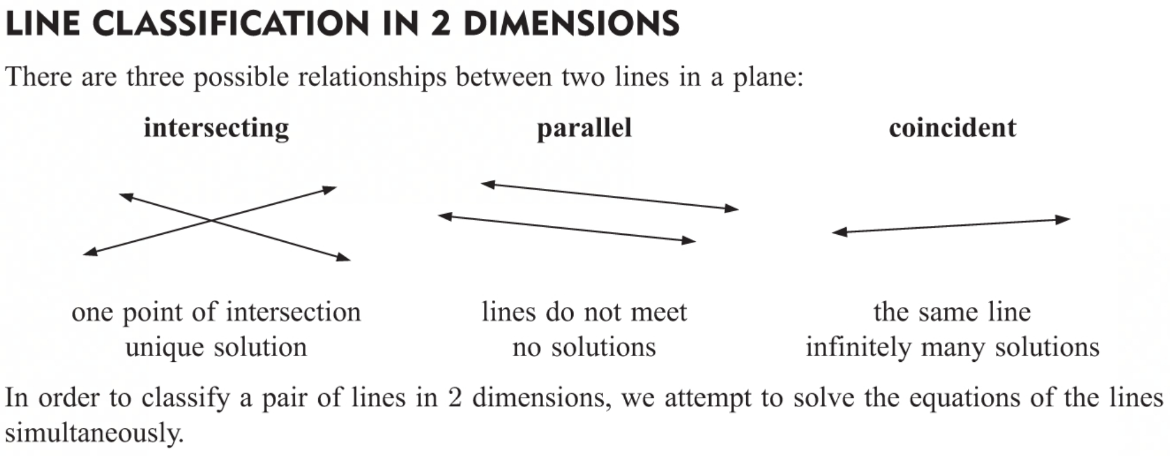
**6E THE SHORTEST DISTANCE FROM A POINT TO A LINE**

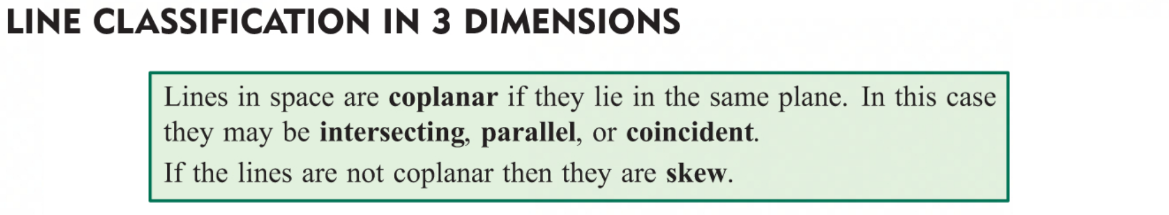
* You can find the shortest distance from the point with position vector to the line with equation using the following method:
* Find the value of which solves
* Substitute this value of into the equation of the line to find the coordinates of the point on the line, , which is closest to
* Calculate

**6F INTERSECTING LINES**

* Vector equations of two intersecting lines can be **solved simultaneously** to find the point where the lines meet.

**6G RELATIONSHIPS BETWEEN LINES**

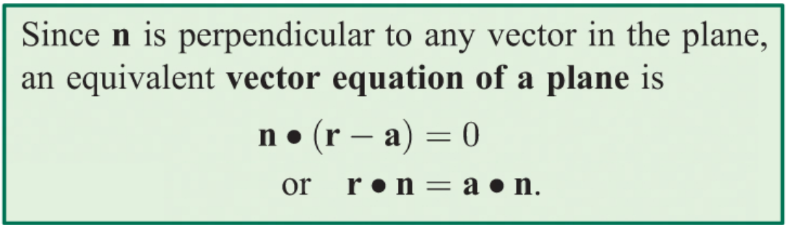


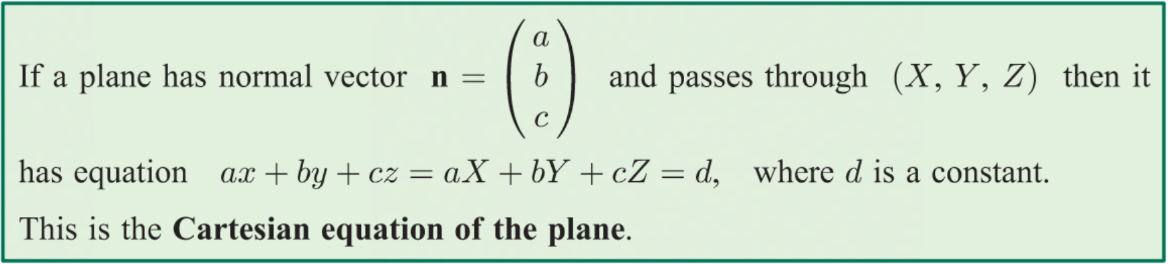


* To find **the shortest distance between parallel lines**, choose any point on one of the lines and find the shortest distance to the other line using the method outlined in section 6E above.
* To find **the shortest distance between skew lines** follow this method:
* We need to find two points and , one on each line, so that is perpendicular to *both* skew lines. So, if the direction vectors of the lines are and respectively, then
* Write in terms of the parameters for the two lines, then use to write a set of three equations in three unknowns. Solve these simultaneously to find the points and
* The shortest distance between the lines is

**6H PLANES**







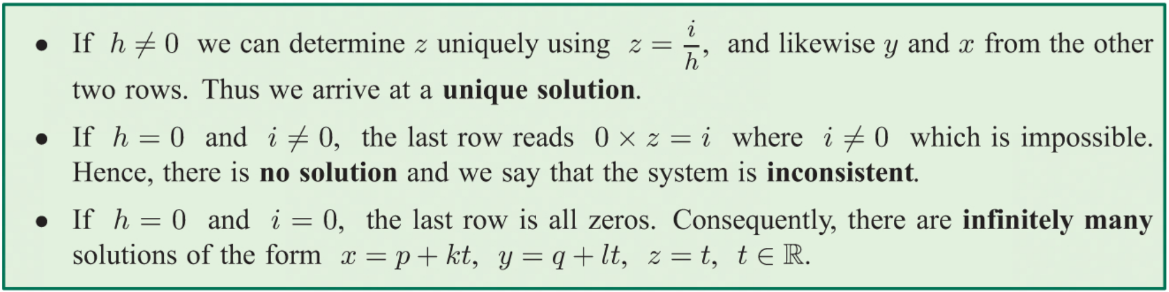
* The **distance from a point to a plane** is

**6I ANGLES IN SPACE**

* The **acute** **angle**, , **between a line** with direction vector,  **and a plane** with normal vector, , is
* The **acute** **angle**, , **between two planes** with normal vectors, and is

**6J SOLVING 3 × 3 LINEAR SYSTEMS**

* Write the system in **augmented matrix form**
* Use row operations to reduce it to **echelon** **form**



**6K INTERSECTING PLANES**

