**Stage 2 Specialist Mathematics**

**Functions and Sketching Graphs Test**

**Topic 3: Subtopics 3.1, 3.2, 3.3**

**Total Marks - 45**

**(Calculator not permitted. One A4 page of handwritten notes permitted.)**

1. (9 marks)

Let $f\left(x\right)=\sqrt{x+3}$ and $g\left(x\right)=x^{2}-4x$.

1. State the domain of $f\left(x\right)$.

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(1 mark)

1. Accurately draw the graph of $y=g\left(x\right)$ on the axes below:

$$x$$

$$y$$

$$-1$$

$$1$$

$$2$$

$$3$$

$$4$$

$$5$$

$$-6$$

$$-4$$

$$-2$$

$$2$$

$$4$$

$$6$$

(2 marks)

* 1. Hence state the range of $g\left(x\right)$.

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(1 mark)

1. Find the composite function $\left(f∘g\right)\left(x\right)$.

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(1 mark)

* 1. Solve the equation $g\left(x\right)=-3$.

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(2 marks)

* 1. Hence, using your answer to (c) (ii) above and your graph from (b) (i), state the values of $x$ for which $\left(f∘g\right)\left(x\right)$ exists.

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(2 marks)

1. (9 marks)

The figure below shows the graph of $g\left(x\right)=2\tan(x)$, where $-π\leq x\leq π$, $x\ne \pm \frac{π}{2}$.

$$x$$

$$g\left(x\right)$$

$$-π$$

$$-\frac{π}{2}$$

$$\frac{π}{2}$$

$$π$$

$$-3$$

$$-2$$

$$-1$$

$$1$$

$$2$$

$$3$$

1. Explain why $g\left(x\right)$ is a function but does not have an inverse function.

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(2 marks)

1. Explain why the following function *does* have an inverse function:

$f\left(x\right)=2\tan(x)$, where $-\frac{π}{2}<x<\frac{π}{2}$.

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(1 mark)

* 1. Show that $f^{-1}\left(x\right)=\arctan(\frac{x}{2})$

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(2 marks)

* 1. On the axes below, sketch $f^{-1}\left(x\right)=\arctan(\frac{x}{2})$.

$$x$$

$$f^{-1}\left(x\right)$$

$$-3$$

$$-2$$

$$-1$$

$$1$$

$$2$$

$$3$$

$$\frac{π}{2}$$

$$-\frac{π}{2}$$

(2 marks)

* 1. State the domain and range of $f^{-1}\left(x\right)$ in exact form.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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(2 marks)

1. (9 marks)
2. Accurately draw the graph of $y=\left(x+1\right)\left(x-5\right)$ on the axes below:

$$x$$

$$y$$

$$-8$$

$$-6$$

$$-4$$

$$-2$$

$$2$$

$$4$$

$$6$$

$$8$$

$$-8$$

$$-6$$

$$-4$$

$$-2$$

$$2$$

$$4$$

$$6$$

$$8$$

(2 marks)

1. Let $f\left(x\right)=\frac{5}{\left(x+1\right)\left(x-5\right)}$.
	1. Draw the graph of $y=f\left(x\right)$ on the axes below:

Clearly show the behaviour of the function near the asymptote(s).

$$x$$

$$y$$

$$-8$$

$$-6$$

$$-4$$

$$-2$$

$$2$$

$$4$$

$$6$$

$$8$$

$$-8$$

$$-6$$

$$-4$$

$$-2$$

$$2$$

$$4$$

$$6$$

$$8$$

(3 marks)

* 1. Draw the graph of $y=\left|f\left(x\right)\right|$ on the axes below:

Clearly show the behaviour of the function near the asymptote(s).

$$x$$

$$y$$

$$-8$$

$$-6$$

$$-4$$

$$-2$$

$$2$$

$$4$$

$$6$$

$$8$$

$$-8$$

$$-6$$

$$-4$$

$$-2$$

$$2$$

$$4$$

$$6$$

$$8$$

(1 mark)

* 1. Draw the graph of $y=f\left(\left|x\right|\right)$ on the axes below:

Clearly show the behaviour of the function near the asymptote(s).

$$x$$

$$y$$

$$-8$$

$$-6$$

$$-4$$

$$-2$$

$$2$$

$$4$$

$$6$$

$$8$$

$$-8$$

$$-6$$

$$-4$$

$$-2$$

$$2$$

$$4$$

$$6$$

$$8$$

(3 marks)

1. (9 marks)

Let $f\left(x\right)=\frac{x^{2}-8x+16}{2x-4}$. $\left[Bonus information: f'\left(x\right)=\frac{2x\left(x-4\right)}{\left(2x-4\right)^{2}}\right]$

1. Use synthetic division to write $f\left(x\right)$ in the form $f\left(x\right)=q\left(x\right)+\frac{r\left(x\right)}{\left(x-α\right)}$.

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(3 marks)

1. Hence sketch the graph of $y=f\left(x\right)$ on the axes below.

Clearly show the axes intercepts and the behaviour of the function near the asymptote(s).

$$x$$

$$y$$

$$-7$$

$$-6$$

$$-5$$

$$-4$$

$$-3$$

$$-2$$

$$-1$$

$$1$$

$$2$$

$$3$$

$$4$$

$$5$$

$$6$$

$$7$$

$$-8$$

$$-6$$

$$-4$$

$$-2$$

$$2$$

$$4$$

$$6$$

$$8$$

(6 marks)

1. (9 marks)

Let $f\left(x\right)=\frac{2x^{2}}{x^{2}+x-2}$. $\left[Bonus information: f'\left(x\right)=\frac{2x\left(x-4\right)}{\left(x^{2}+x-2\right)^{2}}\right]$

1. Use polynomial division and factorisation to write $f\left(x\right)$ in the form $f\left(x\right)=q\left(x\right)+\frac{r\left(x\right)}{\left(x-α\right)\left(x-β\right)}$.

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(3 marks)

1. Hence sketch the graph of $y=f\left(x\right)$ on the axes below.

Clearly show axes and other intercepts and the behaviour of the function near the asymptote(s).

$$x$$

$$y$$

$$-7$$

$$-6$$

$$-5$$

$$-4$$

$$-3$$

$$-2$$

$$-1$$

$$1$$

$$2$$

$$3$$

$$4$$

$$5$$

$$6$$

$$7$$

$$-4$$

$$-3$$

$$-2$$

$$-1$$

$$1$$

$$2$$

$$3$$

$$4$$

(6 marks)