**Stage 2 Specialist Mathematics**

**Functions and Sketching Graphs Test**

**Topic 3: Subtopics 3.1, 3.2, 3.3**

**Total Marks - 45**

**(Calculator not permitted. One A4 page of handwritten notes permitted.)**

1. (9 marks)

Let and .

1. State the domain of .

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(1 mark)

1. Accurately draw the graph of on the axes below:

(2 marks)

* 1. Hence state the range of .

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(1 mark)

1. Find the composite function .

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(1 mark)

* 1. Solve the equation .

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(2 marks)

* 1. Hence, using your answer to (c) (ii) above and your graph from (b) (i), state the values of for which exists.

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(2 marks)

1. (9 marks)

The figure below shows the graph of , where , .

1. Explain why is a function but does not have an inverse function.

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(2 marks)

1. Explain why the following function *does* have an inverse function:

, where .

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(1 mark)

* 1. Show that

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(2 marks)

* 1. On the axes below, sketch .

(2 marks)

* 1. State the domain and range of in exact form.

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(2 marks)

1. (9 marks)
2. Accurately draw the graph of on the axes below:

(2 marks)

1. Let .
   1. Draw the graph of on the axes below:

Clearly show the behaviour of the function near the asymptote(s).

(3 marks)

* 1. Draw the graph of on the axes below:

Clearly show the behaviour of the function near the asymptote(s).

(1 mark)

* 1. Draw the graph of on the axes below:

Clearly show the behaviour of the function near the asymptote(s).

(3 marks)

1. (9 marks)

Let .

1. Use synthetic division to write in the form .

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(3 marks)

1. Hence sketch the graph of on the axes below.

Clearly show the axes intercepts and the behaviour of the function near the asymptote(s).

(6 marks)

1. (9 marks)

Let .

1. Use polynomial division and factorisation to write in the form .

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(3 marks)

1. Hence sketch the graph of on the axes below.

Clearly show axes and other intercepts and the behaviour of the function near the asymptote(s).

(6 marks)