

Question 9 (15 marks)

Consider $f(x) = \frac{x^2 - 1}{x + 2}$.

(a) Show that $f(x) = x - 2 + \frac{3}{x + 2}$.

$x - 2 + \frac{3}{x + 2} = \frac{(x - 2)(x + 2)}{x + 2} + \frac{3}{x + 2}$
$= \frac{x^2 - 4 + 3}{x + 2}$
$= \frac{x^2 - 1}{x + 2}$

(1 mark)

(b) Sketch the graph of $y = f(x)$ on Figure 7 below.
Clearly label all asymptotes and the axes intercepts.

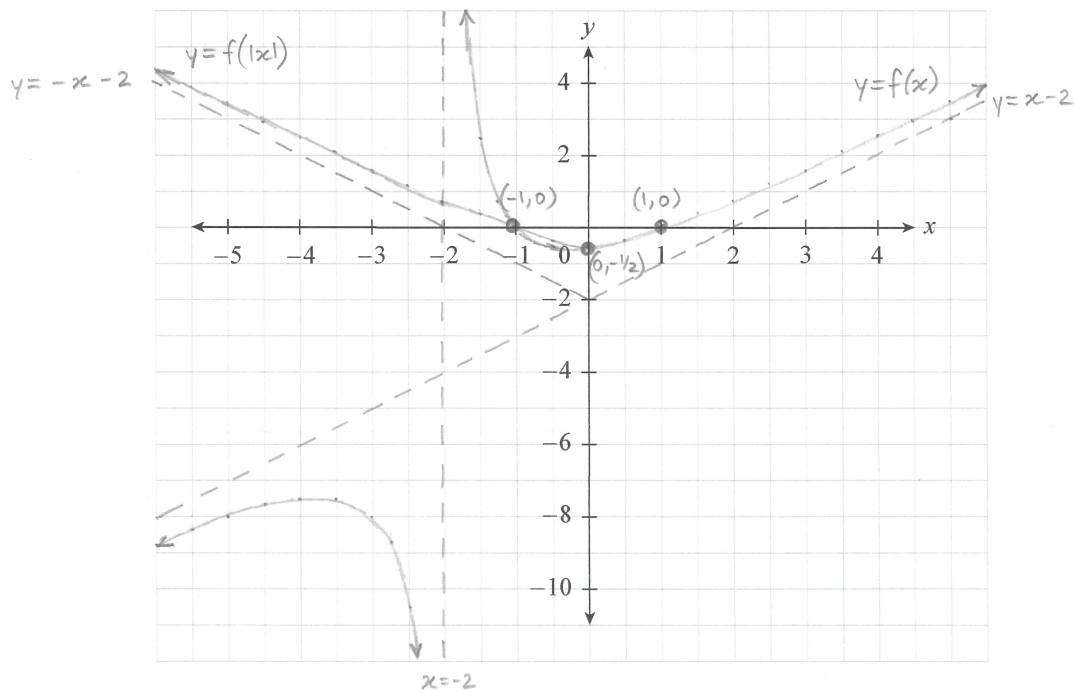


Figure 7

(4 marks)

(c) (i) On Figure 7 above, sketch and clearly label the graph of $y = f(|x|)$.

(2 marks)

(ii) State the interval for which $f(|x|) > f(x)$ for $x > -2$.

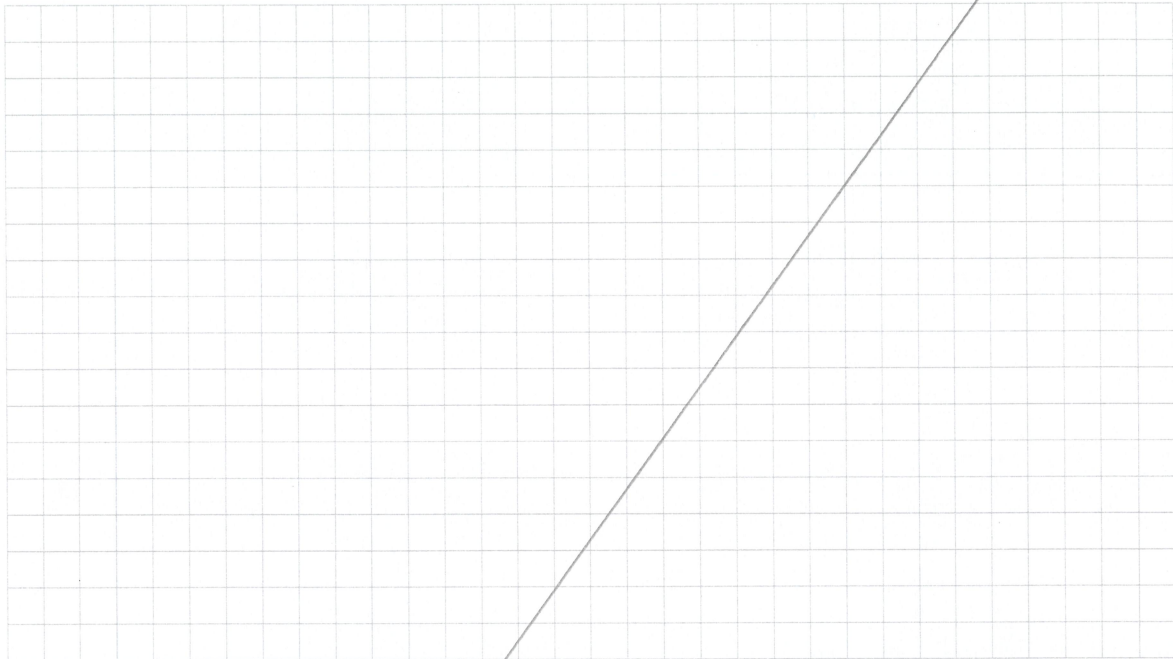
$-1 < x < 0$

(1 mark)

(d) (i) Show that the expression for finding the area between $f(|x|)$ and $f(x)$ for $x > -2$ is given by

$$\int_{-1}^0 -2x + \frac{6x}{4-x^2} dx.$$

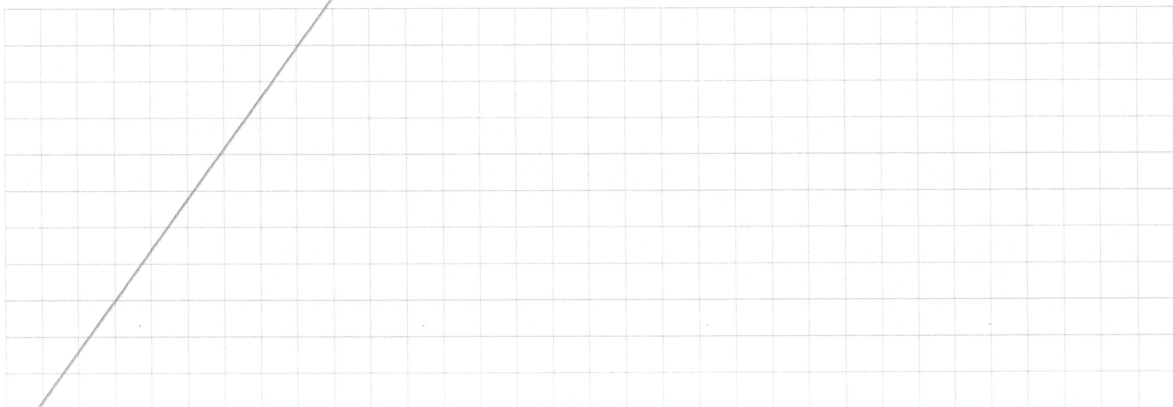
Note that $|x| = -x$ for $x \leq 0$.



(4 marks)

(ii) Hence show that the exact value of the area between $f(|x|)$ and $f(x)$ is

$$1 + 3 \ln\left(\frac{3}{4}\right).$$



(3 marks)