Question 10 (13 marks)

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The rate of change of the number of cars that travel through a particular intersection, per hour, on a typical Monday can be modelled by the function

$$C(t) = -25\cos\left(\frac{\pi}{12}t\right) - 30\cos\left(\frac{\pi}{6}t\right) + 65,$$

where t represents the number of hours since the beginning of Monday.

Figure 12 shows the graph of the model $y = C(t)$ for $0 \le t \le 24$.

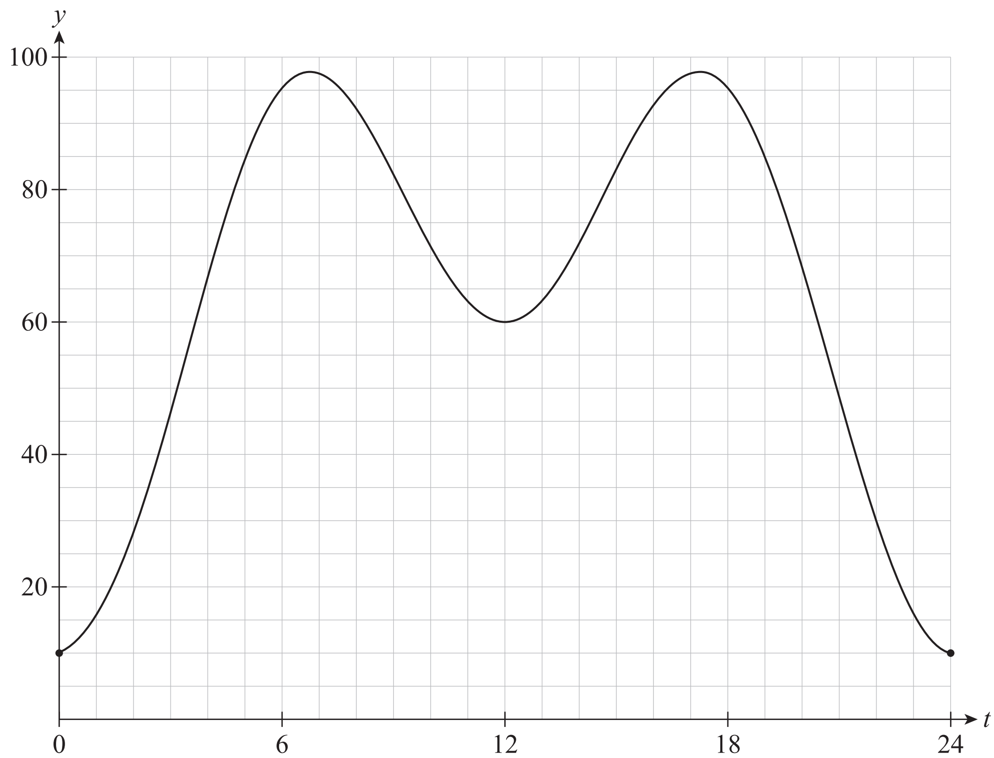

Figure 12

(a) Use the mathematical model to predict, for a Monday:

(i) the rate of change of the number of cars that travel through the intersection at 5 am (i.e. where $t = 5$).

(1 mark)

(ii) the value(s) of t for which the rate of change of the number of cars that travel through the intersection will be at a maximum.

(1 mark)

(iii) the total number of cars that will travel through the intersection between 7 am and 1 pm.

(2 marks)

(b) On the axes in Figure 12, represent the quantity that you calculated in part (a)(iii). (1 mark)

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(c) The rate of change of the number of cars that travel through the intersection per hour is approximately the same on Mondays and Tuesdays. Figure 13 shows the graph of the model $y = C(t)$ for $0 \le t \le 48$. This model represents the rate of change of the number of cars that travel through the intersection per hour on a typical Monday and Tuesday.

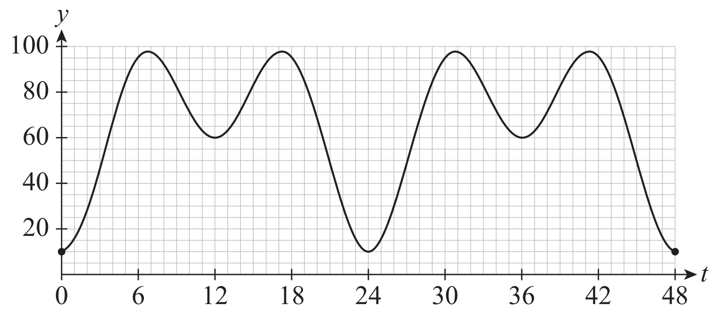

Figure 13

The number of cars that travel through the intersection during any 12-hour period commencing at $t = a$ can be modelled by the function

$$N(a) = \int_a^{a+12} \left(-25\cos\left(\frac{\pi}{12}t\right) - 30\cos\left(\frac{\pi}{6}t\right) + 65\right) \mathrm{d}t.$$

Maintenance is needed on the intersection. During the maintenance, cars will be unable to travel through the intersection. This maintenance takes 12 hours to complete and must commence at $t = a$ on a Monday, hence $0 \le a \le 24$.

(i) Evaluate the integral above to show that $N(a) = \frac{600}{\pi}\sin\left(\frac{\pi}{12}a\right) + 780$.

Note: $\sin(bx + \pi) = -\sin(bx)$ and $\sin(bx + 2\pi) = \sin(bx)$, where b is a real constant.

(4 marks)

(ii) Hence using an algebraic process, find the time on a Monday at which the maintenance should commence in order to minimise the number of cars that will be unable to travel through the intersection.

(4 marks)