Question 5 (6 marks)

Figure 3 shows the graph of $y = f(x)$, where

$$f(x) = \frac{1}{2}e^{3-x} - \frac{1}{2}e^{x-3} + 6.$$

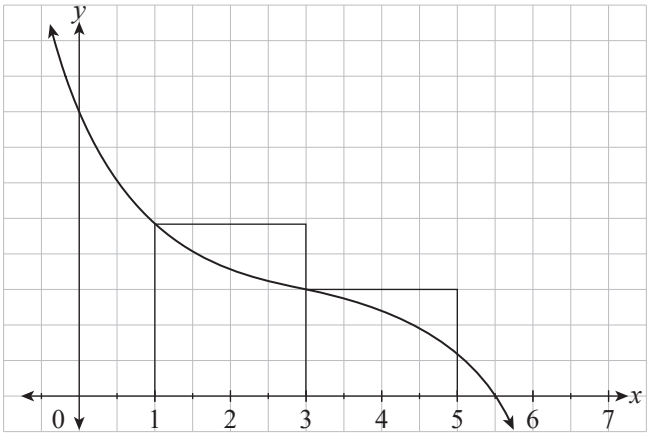

Figure 3

Estimates can be used to approximate A, the area bounded by $y = f(x)$, the x-axis, and the vertical lines $x = 1$ and $x = 5$.

(a) An upper estimate, U, of A can be calculated using the areas of two rectangles of equal width, as shown on Figure 3.

Find the *exact* value of U.

(3 marks)

A lower estimate, L, of A can be calculated using the areas of two rectangles of equal width, as shown on Figure 4.

Figure 4

(b) The *exact* value of L is $e^{-2} - e^2 + 24$.

Show that the average of U and L is 24.

(1 mark)

(c) Write an integral expression to find the value of A.

(1 mark)

Question 5 continues on page 12.

It is known that:

- A (the area bounded by $y = f(x)$, the x-axis, and the vertical lines $x = 1$ and $x = 5$) is 24 square units
- the average of U (the upper estimate) and L (the lower estimate) is 24 square units.

The areas D_1, D_2, D_3, and D_4 are shown shaded on Figure 5, where

- $D_1 + D_2$ is the difference between U and A
- $D_3 + D_4$ is the difference between L and A.

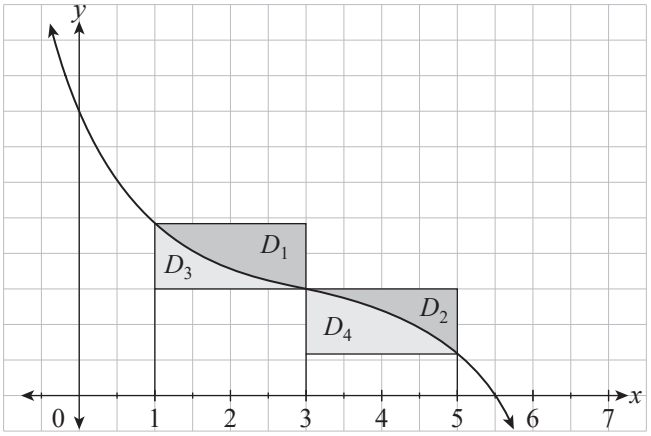

Figure 5

(d) State a relationship between D_1, D_2, D_3, and D_4 if both A, and the average of U and L, are equal to 24.

(1 mark)