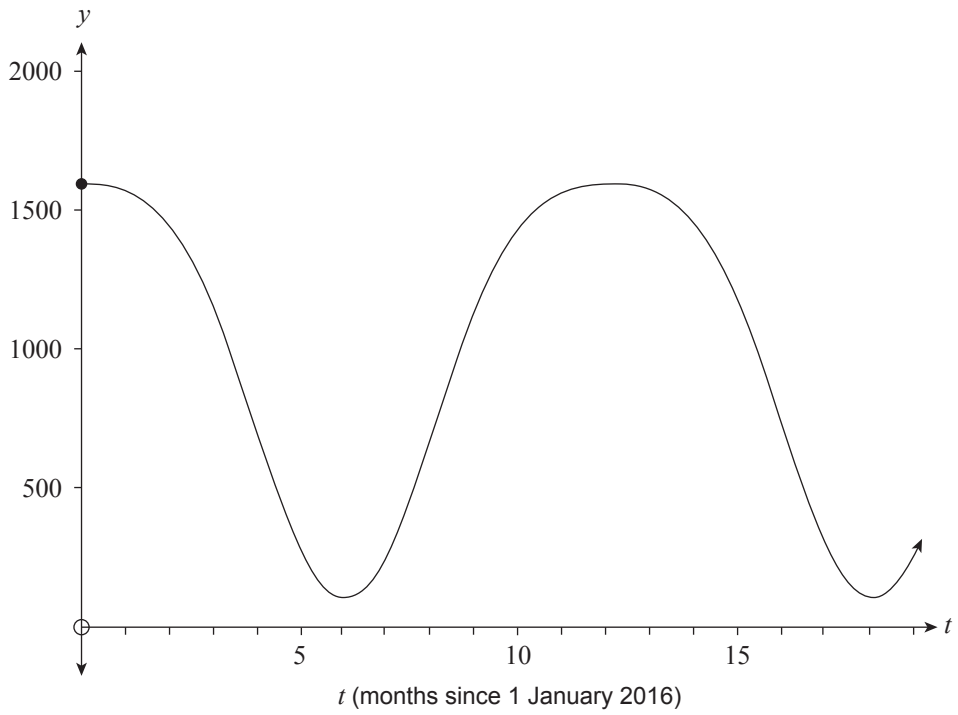
Question 11 (13 marks)

A major city has a wet season and a dry season. A manufacturer in this city produces umbrellas and sells these umbrellas throughout the year. The rate of sales of these umbrellas is not constant. Based on historical sales data, the rate at which the umbrellas are sold can be modelled by the function

$$f(t) = 750\cos\left(\frac{\pi}{6}t\right) + 150\cos\left(\frac{\pi}{3}(t-3)\right) + 1000,$$

where t represents the time, in months, since 1 January 2016.

The graph of $y = f(t)$ is shown below.



(a) For what value of t is the rate of sales of umbrellas first at a minimum?

(1 mark)

(b) (i) Calculate how many umbrellas were sold during the first 3 months of 2016.

(2 marks)

(ii) On the graph above, represent the quantity that you calculated in part (b)(i). (1 mark)

The manufacturer produces 1000 umbrellas per month.

(c) Represent this information on the graph of $y = f(t)$ on page 6. (1 mark)

When the manufacturer produces more umbrellas than they sell, the surplus umbrellas are stored in a warehouse. When the manufacturer produces fewer umbrellas than they sell, the umbrellas that are stored in the warehouse are sold.

(d) When $t = 10$, which *one* of the following is happening? Tick the appropriate box.

The number of umbrellas stored in the warehouse is *increasing*. ☐

The number of umbrellas stored in the warehouse is *decreasing*. ☐ (1 mark)

(e) Show that the manufacturer sold the same number of umbrellas as they produced over the 12 months of 2016.

(2 marks)

(f) During 2016, for what values of t were the number of umbrellas stored in the warehouse increasing?

(2 marks)

(g) At $t = 0$, the warehouse contained 5250 umbrellas.

During 2016, what was the greatest number of umbrellas stored in the warehouse at one time?

(3 marks)