Question 6 (7 marks)

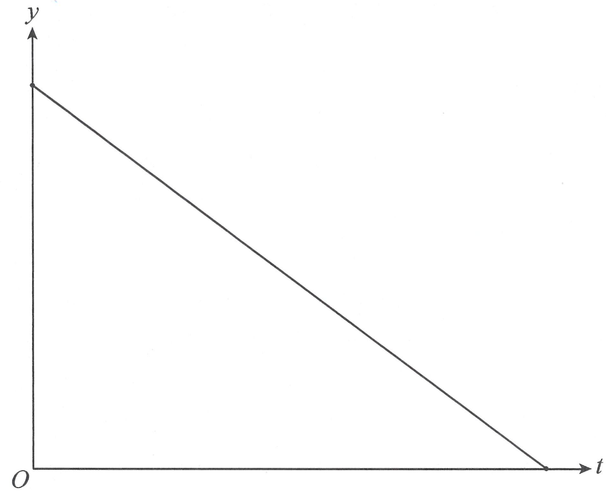
Under normal road conditions, the velocity of a motor vehicle, once its brakes 'lock', decreases at a constant rate of $6\ \text{m s}^{-2}$ until the vehicle stops.

The velocity of a vehicle t seconds after its brakes become locked can be modelled by the function

$$v(t) = -6t + k\ \text{m s}^{-1},$$

where k is a real number representing the initial velocity of the vehicle.

The graph of $y = v(t)$ is shown below.



The graph of $y = v(t)$ has a horizontal axis intercept.

(a) Find the coordinates of this intercept, in terms of k.

$v(t) = 0 \Rightarrow -6t + k = 0$

$-6t = -k$

$t = \frac{k}{6}$ seconds

(2 marks)

(b) (i) Write down an integral expression for d, the distance travelled by a vehicle from $t = 0$ until it stops.

$d = \int_0^{k/6} v(t)\, dt$ metres

(1 mark)

(ii) Complete this integration to find d, in terms of k.

$d = \int_0^{k/6} -6t + k\, dt$

$= \left[-3t^2 + kt \right]_0^{k/6}$

$= -\frac{k^2}{12} + \frac{k^2}{6}$

$= \frac{k^2}{12}$ metres

(2 marks)

(c) A particular vehicle is travelling under normal road conditions. Once its brakes become locked, the vehicle travels 20 metres before stopping.

Using your answer to part (b)(ii), find k. Give your answer correct to one decimal place.

$d = 20 \Rightarrow \frac{k^2}{12} = 20$

$k^2 = 240$

$k = \sqrt{240}$

$= 15.5 \text{ ms}^{-1}$

(2 marks)