

Question 7 (8 marks)

(a) Determine $g'(x)$, given that $g(x) = \frac{(\ln x)^2}{2}$.

$$g'(x) = \frac{2 \ln x}{2} \cdot \frac{1}{x}$$

$$= \frac{\ln x}{x}$$

(2 marks)

(b) The value of $A(k)$ is given by

$$A(k) = \int_1^e \frac{\ln x + k}{x} dx,$$

where k is a positive integer.

(i) Complete the table below by evaluating $A(k)$ for $k = 2$ and $k = 3$.

k	1	2	3
$A(k)$	1.5	2.5	3.5

(1 mark)

(ii) Hence make a conjecture about $A(k)$ for any value of k .

$$A(k) = k + 0.5$$

(1 mark)

(iii) Prove or disprove your conjecture from part (b)(ii).

$$\begin{aligned} A(k) &= \int_1^e \frac{\ln x + k}{x} dx \\ &= \int_1^e \frac{\ln x}{x} + \frac{k}{x} dx \\ &= \left[\frac{(\ln x)^2}{2} + k \ln x \right]_1^e \quad \text{using part (a) above} \\ &= \left(\frac{(\ln e)^2}{2} + k \ln e \right) - \left(\frac{(\ln 1)^2}{2} + k \cdot \ln 1 \right) \\ &= \frac{1}{2} + k \quad [\text{proved}] \end{aligned}$$

(4 marks)