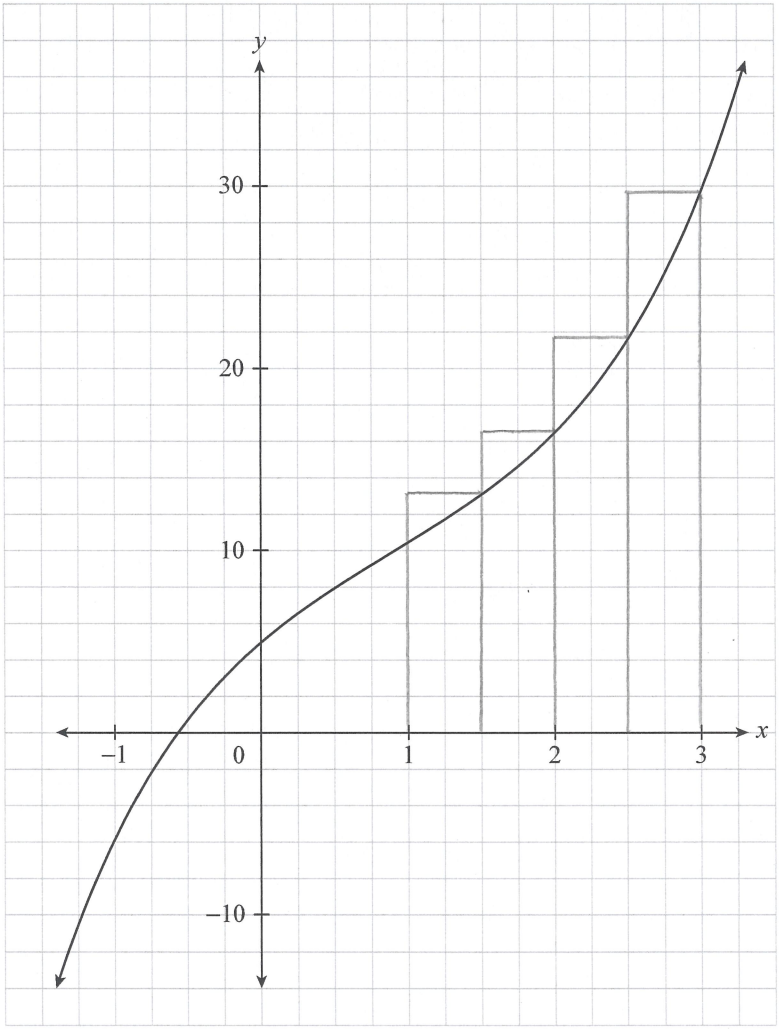
Question 11 (10 marks)

Consider the function $f(x) = e^x + 10 - 6e^{-x}$. The graph of $y = f(x)$ is shown below.



(a) An estimate is required for A, the area between the graph of $y = f(x)$ and the x-axis from $x = 1$ to $x = 3$.

(i) An overestimate of area A is to be calculated, using four rectangles of equal width.
On the graph above, draw the four rectangles used to obtain this overestimate. (1 mark)

(ii) Calculate this overestimate, correct to four significant figures.

Overestimate $= 0.5 \times f(1.5) + 0.5 \times f(2) + 0.5 \times f(2.5) + 0.5 \times f(3)$

$= 40.60$ units^2

(2 marks)

(b) (i) Determine $f''(x)$.

$f'(x) = e^x + 6e^{-x}$

$f''(x) = e^x - 6e^{-x}$

(2 marks)

(ii) The solution to $f''(k) = 0$, correct to three significant figures, is $k = 0.896$.

Determine the exact value of k.

$f''(x) = 0 \Rightarrow e^x - 6e^{-x} = 0$

$e^x = 6e^{-x}$

$e^{2x} = 6$

$2x = \ln 6$

$x = \frac{1}{2}\ln 6$

(2 marks)

(iii) For what values of x is $f''(x) > 0$?

$x > \frac{1}{2}\ln 6$

(1 mark)

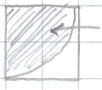
(c) An underestimate of area A could also be calculated, using four rectangles of equal width.

With reference to your answer to part (b)(iii), explain which estimate — this underestimate or the overestimate calculated in part (a)(ii) — would be closer to area A. Do ***not*** calculate this underestimate.

From (b)(iii) above, $f(x) > 0$ when $x > \frac{1}{2}\ln 6$ ie. when $x > 0{\cdot}896$

$\therefore f(x)$ is concave upwards for $1 \leq x \leq 3$


error obtained using underestimate


error obtained using overestimate

So, the underestimate would be closer to area A

(2 marks)