

Question 5 (6 marks)

Figure 3 shows the graph of $y = f(x)$, where

$$f(x) = \frac{1}{2}e^{3-x} - \frac{1}{2}e^{x-3} + 6.$$

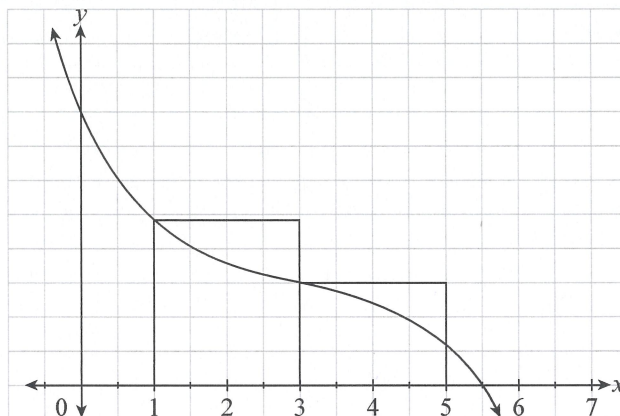


Figure 3

Estimates can be used to approximate A , the area bounded by $y = f(x)$, the x -axis, and the vertical lines $x = 1$ and $x = 5$.

- (a) An upper estimate, U , of A can be calculated using the areas of two rectangles of equal width, as shown on Figure 3.

Find the *exact* value of U .

$$\begin{aligned} U &= 2 \times f(1) + 2 \times f(3) \\ &= e^2 - e^{-2} + 12 + e^0 - e^0 + 12 \\ &= e^2 - e^{-2} + 24 \text{ units}^2 \end{aligned}$$

(3 marks)

A lower estimate, L , of A can be calculated using the areas of two rectangles of equal width, as shown on Figure 4.

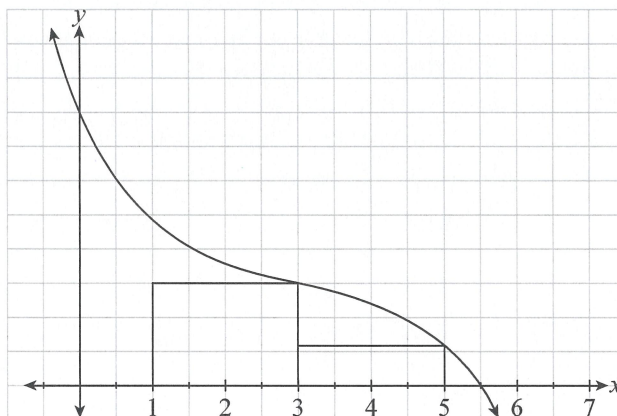


Figure 4

(b) The exact value of L is $e^{-2} - e^2 + 24$.

Show that the average of U and L is 24.

$$\frac{U+L}{2} = \frac{e^2 - e^{-2} + 24 + e^{-2} - e^2 + 24}{2}$$

$$= 24$$

(1 mark)

(c) Write an integral expression to find the value of A .

$$A = \int_1^5 f(x) dx$$

(1 mark)

Question 5 continues on page 12.

